Weakly Sequential Completeness of Orlicz Spaces*

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Banach space X is called to be weakly sequential complete space, if for each weak Cauchy Sequence $\{x_n\}$ in X, there exists a element y in X such that $x_n \xrightarrow{w} x$ $(n \to \infty)$.

Weakly sequential completeness in close relationship with refrexivity、separability、weak convexity、bases and isomorphic subspaces in Banach spaces.

In this paper, the weakly sequential completeness of Orlicz spaces is investigated. The sufficient and necessary condition for a Orlicz space to be weakly sequential complete is obtained. The application of the condition to the problem of isomorphic subspace of Orlicz spaces leads to finding a new character of reflexivity of Orlicz spaces.

Let $M(u)$ be a continuous Yong function, $G \subset R^n$ a bounded closed set. The main results are following.

Theorem 1 Orlicz space $L^*_M(G)$ is weakly sequential complete iff $M(u)$ satisfies Δ_2-condition for large u.

Theorem 2 Orlicz space $L^*_M(G)$ is reflexive iff no closed subspace of L $L^*_M(G)$ is isomorphic to C_0 or l^1.

Corollary 1 If Orlicz space $L^*_M(G)$ with Luxemburg norm $\|\cdot\|_{(M)}$ is weakly uniform convex, then $L^*_M(G)$ is refrexive.

Corollary 2 If Orlicz space L^*_M with Orlicz norm $\|\cdot\|_M$ (or Luxemburg norm $\|\cdot\|_{(M)}$ is uniformly non-$l_n^{(1)}$ space[3] for some $n \geq 2$, then $L^*_M(G)$ is refrexive.

References

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