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A Note on Pseudo Ideals of Fields*

Zhou Dingxuan

(Dept. Math.,Zhejiang University, Hangzhou)

Definition A subring A of a ring R is called a pseudo ideal, if for any $r\in R$ and $a\in A$, $r^2a\in A$ and $ar^2\in A$. The subring $(R)^2=\{\Sigma\pm r_1^2\cdots r_t^2|r_i\in R,\ t\in N\}$ is called the square closed ring of R.

Lemma[2] The square closed ring of a division ring D is the intersection of all nontrivial pseudo ideals of D.

Theorem 1 If a simple ring R with a unit has a nontrivial pseudo ideal A containing an invertible element, R has characteristic $p=2$. In particular, the necessary condition for a division ring containing a nontrivial pseudo ideal is $p=2$.

Proof Suppose $a\in A$ is invertible, $a^2\in A$, hence $e=(a^{-1})^2*a^2\in A$ and $\{x^2|x\in R\}\subset A$.

For any $x\in R$, $2x=(x+e)^2-x^2-e\in A$. We have $I=\{2x|x\in R\}\subset A\neq R$. Now I is an ideal of R, and $I\neq R$, but R is a simple ring, we have $\{2x|x\in R\}=\{0\}$, that is, $p=2$.

By [2], a noncommutative division ring has no proper pseudo ideals, we have the following theorem:

Theorem 2 A division ring R has a proper pseudo ideal if and only if (1) R is a field; (2) R has characteristic $p=2$; (3) $\{x^2|x\in R\}\neq R$.

Proof ($\Rightarrow$) Suppose A is a proper pseudo ideal of R, $0\subsetneq A\subsetneq R$, we know[2] that R is a field.

By Theorem 1, we have Char $R=2$.

(3) is trivial since $\{x^2|x\in R\}\subset A\neq R$.

($\Leftarrow$) It is sufficient for us to prove that $A=\{x^2|\ x\in R\}$ is a pseudo ideal if Char $R=2$ and R is a field:

For any $x^2, y^2\in A$, $x^2-y^2=(x+y)^2\in A$.

For any $r\in R$ and $x^2\in A$, $x^2*r^2=(xr)^2\in A$; $r^2*x^2=(rx)^2\in A$.

The proof is complete.

Note that a finite field F with Char $F=2$ satisfies $F=\{r^2|r\in F\}$, we have

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The Best Simultaneous Approxlmation

Li Jiangbo

(Zhejiang, Lishui Teachers' College)

Abstract

In this paper, we study the problem of simultaneous approximation to bivariate function by one variate function, i.e., minimizing the expression

$$\max_{x\in X}\left(\int_Y |f(x,y)-g(x)|^p d\mu\right)^{\frac{1}{p}} \quad (p\geqslant 1)$$

over G, G is an noempty subset in $C(X)$.

We obtain the characterization theorems, the uniqueness theorems, the strong uniqueness theorems and de la Vallée Poussin theorems. We also establish the first algorithm of Remes type and two limit theorems.

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Theorem 2 in [2] as a corollary:

Corollary A finite field has no proper pseudo ideals.

Thus we almost fail to characterise a division ring by pseudo ideals instead of ideals.

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References

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