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Stability on a Class of Linear Neutral Differential Systems with Distributed Argument *

BAO Jun-dong[1], *HU Yong-zhen*[1], *AO Deng*[2]

(1. Dept. of Math., Inner Mongolia Normal University, Huhhot 010022, China;
2. Dept. of Pharm., Inner Mongolia Medicine College, Huhhot 010059, China)

Abstract: In this paper, a class of linear neutral differential system with distributed delay is considered. Sufficient conditions for the zero solution of the system to be uniformly stable as well as asymptotically stable are obtained.

Key words: linear neutral differential system; distributed delay; uniform stability; asymptotic stability.

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1. Introduction

Consider the linear neutral differential system with distributed argument

$$\frac{\mathrm{d}}{\mathrm{d}t}[x_i(t)-\int_{a_i}^{b_i}x_i(t-\theta)\mathrm{d}\alpha_i(t,\theta)]+\sum_{j=1}^{n}\int_{c_{ij}}^{d_{ij}}x_j(t-s)\mathrm{d}\beta_{ij}(t,s)=0,\quad i=1,2,\cdots,n,\qquad(1)$$

where $a_i,b_i,c_{ij},d_{ij}\in(0,\infty),(i,j=1,2,\cdots,n)$, are nonnegative constants, $a_i<b_i,c_{ij}<d_{ij}$, $\alpha_i(t,\theta)$ and $\beta_{ij}(t,s)$, (i,j=1,2,...,n), are continuous functions for every fixed θ and s in $t\ge t_0>0$, and for every $t\in[t_0,\infty)$ they are positive bounded variations in θ and s.

In recent years, many authors (*see*[1 – 7]) have probed the problem of stability of zero solution of first order neutral and delay differential equations

$$\frac{\mathrm{d}}{\mathrm{d}t}[x(t)+p(t)x(t-\tau)]+Q(t)x(t-\delta)=0\qquad(2)$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t)+Q(t)x(t-\delta)=0.\qquad(3)$$

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Biography: BAO Jun-dong (1958-), male, born in Inner Mongolia, currently an associate professor at Inner Mongolia Normal University.

In this paper, we will extend the discussion to the system (1) and develope the results in [1] to system (1).

Denote $N=\{1,2,\cdots,n\}$, $\delta_{ij}=d_{ij}-c_{ij}$,

$$r=\min_{i,j\in N}\{b_i-a_i,d_{ij}-c_{ij}\},\ \rho=\max_{i,j\in N}\{b_i-a_i,d_{ij}-c_{ij}\},\ \delta=\max_{i,j\in N}\{\delta_{ij}\},$$

and $\bigvee_{t=a}^{b} f(t)$ means the total variation of $f(t)$ on [a,b], and

$$Z_i(t)=x_i(t)-\int_{a_i}^{b_i}x_i(t-\theta)\mathrm{d}\alpha_i(t,\theta),\quad i\in N.$$

2. Main results

Theorem 1 *If*

$$\bigvee_{\theta=a_i}^{b_i}\alpha_i(t,\theta)\le\alpha_i<\frac{1}{2},$$

$$2\alpha_i(2-\alpha_i)+\sum_{j=1}^{n}\int_t^{t+\delta}\bigvee_{s=c_{ij}}^{d_{ij}}\beta_{ij}(\theta,s)\mathrm{d}\theta\le\frac{3}{2},\ t\ge t_0,\ i=1,2,...,n,\tag{4}$$

where $\alpha_i, i\in N$, are constants, then the zero solution of system (1) is uniformly stable.

Proof Let $x_i(t,t_0,\phi_i)$ be the solution of system (1) satisfying the initial condition $x_i(s,t_0,\phi_i)=\phi_i(s)$ for $s\in[t_0-\rho,t_0]$. For convenience, we denote that $x_i(t)=x_i(t,t_0,\phi_i)$. Now choose a positive integer l such that $lr\ge 2\delta$, and for any $\varepsilon>0$, define $\eta_i=(1-\alpha_i)\frac{\varepsilon}{n}/(1+\alpha_i)(2a_i+\frac{2}{5})^l, i=1,2,\cdots,n$. In the following we show that for any ϕ_i and any $\bar{t}\ge t_0$, if $\|\phi_i\|<\eta_i$, then we have

$$|x_i(t,\bar{t},\phi_i)|<\frac{\varepsilon}{n},\quad t\ge\bar{t}.\tag{5}$$

In fact, define $\rho_{i1}=(2\alpha_i+\frac{5}{2})\eta_i,\cdots,\rho_{ik}=(2\alpha_i+\frac{5}{2})\rho_{ik-1}, k=2,3,\cdots,l$. Then $\rho_{ik}=(2\alpha_i+\frac{5}{2})^k\eta_i, k=1,2,\cdots,l, i\in N$, and $\eta_i<\rho_{i1}<\rho_{i2}<\cdots<\rho_{il}<\frac{\varepsilon}{n}$, for $i\in N$. At first, we have that

$$|x_i(t,\bar{t},\phi_i)|<\rho_{ik},\quad t\in[\bar{t}(k-1)r,\bar{t}+kr],\quad k=1,2,\cdots,l.\tag{6}$$

Actually we have that for $t\in[\bar{t}-\rho,\bar{t}+r]$

$$\begin{aligned}|x_i(t)|=|&\int_{a_i}^{b_i}x_i(t-\theta)\mathrm{d}\alpha_i(t,\theta)+x_i(\bar{t}_0)-\int_{b_i}^{a_i}x_i(\bar{t}-\theta)\mathrm{d}\alpha_i(\bar{t},\theta)-\\&\sum_{j=1}^{n}\int_{\bar{t}}^{t}\int_{c_{ij}}^{d_{ij}}x_j(s-\theta)\mathrm{d}\beta_{ij}(s,\theta)\mathrm{d}s|\\&\le\eta_i(2\alpha_i+1)+\sum_{j=1}^{n}\int_{\bar{t}}^{i+r}\bigvee_{\theta=c_{ij}}^{d_{ij}}\beta_{ij}(s,\theta)\mathrm{d}s<\eta_i(2\alpha_i+\frac{5}{2})=\rho_{i1}.\end{aligned}$$

Which shows that (6) holds for $k=1$ and hence

$$|x_i(t)| < \rho_{i1}, \quad \text{for} \quad t \in [\bar{t}-\rho, \bar{t}+r], \quad i=1,2,\cdots,n.$$

Now we replace η_i with ρ_{i1} and repeate the above procedures, we have that

$$|x_i(t)| < [2\alpha_i + \frac{5}{2}]\rho_{i2}, \quad t \in [\bar{t}+r, \bar{t}+2r].$$

Hence by induction we can show that (6) holds.

By contradiction, we assume that (5) is not true. Then by (6) there exists some $T > \bar{t}+lr$ such that $|x_i(T)| = \frac{\varepsilon}{n}$ and $|x_i(t)| < \frac{\varepsilon}{n}$ for $t \in [\bar{t}, T)$. Without loss of generality, we assume that $x_i(T) = \frac{\varepsilon}{n}$, and we have

$$Z_i(T) = x_i(T) - \int_{a_i}^{b_i} x_i(T-\theta)\mathrm{d}\alpha_i(T,\theta) \geq \frac{\varepsilon}{n}(1 - \bigvee_{\theta=\alpha_i}^{b_i} \alpha_i(T,\theta)) \geq \frac{\varepsilon}{n}(1-\alpha_i). \qquad (7)$$

On the other hand, since

$$Z_i(\bar{t}+lr) = x_i(\bar{t}+lr) - \int_{a_i}^{b_i} x_i(\bar{t}+lr-\theta)\mathrm{d}\alpha_i(\bar{t}+lr,\theta)$$
$$< \rho_{il}(1+\alpha_i) = \frac{\varepsilon}{n}(1-\alpha_i) \leq Z_i(T). \qquad (8)$$

Thus by (7) and (8), there exists $\xi \in [\bar{t}+lr, T]$ such that

$$Z_i(\xi) = \max_{\bar{t}+lr<t\leq T} Z_i(t), \quad \text{and} \quad Z_i(\xi) > Z_i(t), \quad \text{for} \quad t \in (\bar{t}+lr, \xi).$$

and

$$x_i(\xi) = Z_i(\xi) + \int_{a_i}^{b_i} x_i(\xi-\theta)d\alpha_i(\xi,\theta) \geq Z_i(T) - \frac{\epsilon}{n}\bigvee_{\theta=a_i}^{b_i} \alpha_i(\xi,\theta)$$
$$\geq Z_i(T) - \alpha_i\frac{\varepsilon}{n} \geq \frac{\varepsilon}{n}(1-\alpha_i) - \alpha_i\frac{\varepsilon}{n} = \frac{\varepsilon}{n}(1-2\alpha_i) > 0.$$

Next, we will show that $x_i(\xi-\delta_{ij}) \leq 0$. Otherwise $x_i(\xi-\delta_{ij}) > 0$, thus there must be a left neighbor of $\xi-\delta_{ij}$, say $(\xi-\delta_{ij}-h, \xi-\delta_{ij})$ for some $h>0$, such that $x_i(t) > 0, t \in (\xi-\delta_{ij}-h, \xi-\delta_{ij})$. This implies that $x_i(t-\delta_{ij}) > 0$, for $t \in (\xi-h, \xi)$, and therefore by system (1) we know that $Z_i(t)$ is not increasing on $(\xi-h,\xi)$, this contridicts the definition of ξ, and so $x_i(\xi-\delta_{ij}) \leq 0$. Hence there exists $T_i \in [\xi-\delta_{ij}, \xi)$ such that $x_i(T_i) = 0$, by system (1) we have

$$Z_i'(t) \leq \sum_{j=1}^{n} \int_{c_{ij}}^{d_{ij}} \frac{\varepsilon}{n}\mathrm{d}\beta_{ij}(t,s) \leq \frac{\varepsilon}{n}\sum_{j=1}^{n} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(t,s), \quad \bar{t} \leq t \leq T. \qquad (9)$$

Since $t \in [T_i, \xi]$ implies that $t-\psi \leq T_i, \psi \in [0, \delta_{ij}]$, so from (9) we have

$$Z_i(T_i) - Z_i(t_i-\psi) \leq \frac{\varepsilon}{n}\sum_{j=1}^{n} \int_{t-\psi}^{T_i} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(\theta,s)\mathrm{d}\theta,$$

that is

$$-x_i(t-\psi) \le -\int_{a_i}^{b_i} x_i(t-\psi-\theta)d\alpha_i(t-\psi,\theta) + \int_{a_i}^{b_i} x_i(T_i-\theta)\mathrm{d}\alpha_i(T_i,\theta) +$$

$$\frac{\varepsilon}{n}\sum_{j=1}^{n}\int_{t-\psi}^{T_i}\bigvee_{s=c_{ij}}^{d_{ij}}\beta_{ij}(\theta,s)\mathrm{d}\theta$$

$$\le \frac{\varepsilon}{n}[2\alpha_i + \sum_{j=1}^{n}\int_{t-\psi}^{T_i}\bigvee_{s=c_{ij}}^{d_{ij}}\beta_{ij}(\theta,s)\mathrm{d}\theta], \quad t\in[T_i,\xi]. \tag{10}$$

Substituting (10) to (1), we obtain that

$$Z_i'(t) \le \frac{\varepsilon}{n}\sum_{j=1}^{n}\int_{c_{ij}}^{d_{ij}}[2\alpha_i + \sum_{k=1}^{n}\int_{t-\psi}^{T_i}\bigvee_{s=c_{ik}}^{d_{ij}}\beta_{ik}(\theta,s)\mathrm{d}\theta]\mathrm{d}\beta_{ij}(t,\psi), \quad t\in[T_i,\xi], \quad \psi\in[0,\delta_{ij}], \tag{11}$$

since $\xi - T_i \le \delta_{ij}$ and (4), we have

$$2\alpha_i(2-\alpha_i) + \sum_{j=1}^{n}\int_{T_i}^{\xi}\bigvee_{s=c_{ik}}^{d_{ik}}\beta_{ik}(\theta,s)\mathrm{d}\theta \le \frac{3}{2}, \quad t\in[T_i,\xi]. \tag{12}$$

Now we show that

$$Z_i(T) \le Z_i(\xi) < (1-2\alpha_i)\frac{\varepsilon}{n} \tag{13}$$

holds, so it leads a contradiction to (7). We devide the proof into two cases.

Case 1 If $2\alpha_i + \sum_{j=1}^{n}\int_{T_i}^{\xi}\bigvee_{s=c_{ij}}^{d_{ij}}\beta_{ij}(\theta,s)\mathrm{d}\theta \le 1$, then integrating (11) from T_i to ξ,

$$Z_i(\xi) \le Z_i(T_i) + \frac{\varepsilon}{n}\sum_{j=1}^{n}\int_{T_i}^{\xi}\int_{c_{ij}}^{d_{ij}}[2\alpha_i + \sum_{k=1}^{n}\int_{t-\psi}^{T_i}\bigvee_{s=c_{ij}}^{d_{ij}}\beta_{ik}(\theta,s)\mathrm{d}\theta]\mathrm{d}\beta_{ij}(t,\psi)\mathrm{d}t$$

$$= -\int_{a_i}^{b_i} x_i(T_i-\theta)d\alpha_i(T_i,\theta) + \frac{\varepsilon}{n}\sum_{j=1}^{n}\int_{T_i}^{\xi}\int_{c_{ij}}^{d_{ij}}[2\alpha_i +$$

$$\sum_{k=1}^{n}(\int_{t-\psi}^{t} - \int_{T_i}^{t})\bigvee_{s=c_{ij}}^{d_{ij}}\beta_{ik}(\theta,s)\mathrm{d}\theta]d\beta_{ij}(t,\psi)\mathrm{d}t$$

$$\le \frac{\varepsilon}{n}\alpha_i + \frac{\varepsilon}{n}\sum_{j=1}^{n}\int_{T_i}^{\xi}\int_{c_{ij}}^{d_{ij}}[\frac{3}{2} - 2\alpha_i(1-\alpha_i) - \sum_{k=1}^{n}\int_{T_i}^{t}\bigvee_{s=c_{ij}}^{d_{ij}}\beta_{ik}(\theta,s)\mathrm{d}\theta]\mathrm{d}\beta_{ij}(t,\psi)\mathrm{d}t$$

$$\le \frac{\varepsilon}{n}[\alpha_i + (\frac{3}{2} - 2\alpha_i(1-\alpha_i))\sum_{j=1}^{n}\int_{T_i}^{t}\bigvee_{s=c_{ij}}^{d_{ij}}\beta_{ij}(t,s)\mathrm{d}t - \frac{1}{2}(\sum_{j=1}^{n}\int_{T_i}^{t}\bigvee_{s=c_{ij}}^{d_{ij}}\beta_{ij}(t,s)\mathrm{d}t)^2].$$

On the other hand, we know that function $\alpha_i + (\frac{3}{2} - 2\alpha_i(1-\alpha_i))x - \frac{1}{2}x^2$ is increasing on $x\in(0,1-2\alpha_i)$. Hence

$$Z_i(\xi) \le \frac{\varepsilon}{n}[\alpha_i + l(\frac{3}{2} - 2\alpha_i(1-\alpha_i))(1-2\alpha_i) - \frac{1}{2}(1-2\alpha_i)^2] < (1-\alpha_i)\frac{\varepsilon}{n}. \tag{14}$$

So the first case is complete.

Case 2 If $2\alpha_i + \sum_{j=1}^n \int_{T_i}^{\xi} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(\theta,s)\mathrm{d}\theta > 1$, since $2\alpha_i < 1$, there must be $T_{i1} \in (T_i, \xi)$ such that

$$2\alpha_i + \sum_{j=1}^n \int_{T_{i1}}^{\xi} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(\theta,s)\mathrm{d}\theta = 1.$$

Integrating (9) from T_i to T_{i1} and (11) from T_{i1} to ξ, then merge them together, we have

$$\begin{aligned}
Z_i(\xi) &\le Z_i(T_i) + \frac{\varepsilon}{n}\sum_{j=1}^n \int_{T_i}^{T_{i1}} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(t,s)\mathrm{d}t + \frac{\varepsilon}{n}\sum_{j=1}^n \int_{T_{i1}}^{\xi} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(t,s)[2\alpha_i + \\
&\quad \sum_{k=1}^n \int_{t-\psi}^{T_i} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ik}(\theta,s)\mathrm{d}\theta]\mathrm{d}t \\
&< \frac{\varepsilon}{n}\alpha_i + \frac{\varepsilon}{n}\sum_{j=1}^n \int_{T_{i1}}^{\xi} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(t,s)\mathrm{d}t \sum_{j=1}^n \int_{T_i}^{T_{i1}} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(\theta,s)\mathrm{d}\theta + \\
&\quad 2\alpha_i\frac{\varepsilon}{n}\sum_{j=1}^n \int_{T_i}^{T_{i1}} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(t,s)\mathrm{d}t + \frac{\varepsilon}{n}\sum_{j=1}^n \int_{T_{i1}}^{\xi} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(t,s)\mathrm{d}t + \\
&\quad 2\alpha_i \sum_{k=1}^n \int_{t-\psi}^{T_i} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ik}(\theta,s)\mathrm{d}\theta \\
&\le \frac{\varepsilon}{n}[(3-4\alpha_i) + 2\alpha_i(-1+2\alpha_i + \sum_{j=1}^n \int_{T_i}^{\xi} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(t,s)\mathrm{d}t) + \\
&\quad \sum_{j=1}^n \int_{T_{i1}}^{\xi} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(t,s)\mathrm{d}t(\frac{3}{2} - 2\alpha_i(2-\alpha_i)) - \sum_{j=1}^n \int_{T_i}^{T_{i1}} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(\theta,s)\mathrm{d}\theta] \\
&\le \frac{\varepsilon}{n}[\alpha_i + 2\alpha_i(\frac{3}{2} - 2\alpha_i(2-\alpha_i))(1-2\alpha_i) + (\frac{3}{2} - 2\alpha_i(2-\alpha_i)) - \frac{1}{2}(1-2\alpha_i)^2] \\
&= (1-\alpha_i)\frac{\varepsilon}{n}.
\end{aligned}$$

That is

$$Z_i(\xi) < (1-\alpha_i)\frac{\varepsilon}{n}. \tag{15}$$

Thus by (14) and (15), we lead to a contradiction to (7). Therefore we obtain that

$$\|x(t)\| = \sum_{i=1}^n |x_i(t)| < \varepsilon, \quad \text{for} \quad t \in [\bar{t}, \infty).$$

So the proof of Theorem 1 is complete.

Theorem 2 *Under the condition (4) in the theorem 1 and if*

$$\sum_{j=1}^n \int_{t_0}^{\infty} \bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(\theta,s)\mathrm{d}\theta = \infty, \quad i = 1, 2, \cdots, n, \tag{16}$$

then the zero solution of system (1) is asymptotically stable.

Proof In view of theorem 1, the zero solution of system (1) is unformly stable, hence there exists an $\eta > 0$ such that $x_i(t) = x_i(t, t_0, \phi)$ is bounded for $\phi \in C([t_0 - \rho, t_0), (-\eta, \eta))$.

Without loss of generality, we assume that $x_i(t), i \in N$, is eventually positive, hence $Z_i(t) = x_i(t) - \int_{a_i}^{b_i} x_i(t-\theta)\mathrm{d}\alpha_i(t,\theta)$ is eventually nonincreasing function. Set

$$K_i = \lim_{t\to\infty} Z_i(t),$$

it is obvious that $K_i \in R$, then

$$\overline{\lim_{t\to\infty}} x_i(t) = K_i + \overline{\lim_{t\to\infty}} \int_{a_i}^{b_i} x_i(t-\theta)\mathrm{d}\alpha_i(t,\theta) \le K_i + \alpha_i \overline{\lim_{t\to\infty}} x_i(t).$$

Thus

$$\overline{\lim_{t\to\infty}} x_i(t) \le \frac{K_i}{1-\alpha_i},$$

this implies that $K_i \ge 0$. If $K_i > 0$, consider the following inequallity

$$\begin{aligned}\underline{\lim_{t\to\infty}} x_i(t) &= K_i + \underline{\lim_{t\to\infty}} \int_{a_i}^{b_i} x_i(t-\theta)\mathrm{d}\alpha_i(t,\theta) \ge K_i - \alpha_i \underline{\lim_{t\to\infty}} x_i(t) \\ &\ge K_i - \alpha_i\frac{K_i}{1-\alpha_i} = \frac{1-2\alpha_i}{1-\alpha_i}K_i = \alpha > 0. \qquad (17)\end{aligned}$$

By (17), we conclude that there is a large enough T such that $x_i(t) > \frac{\alpha}{2}$ for $t \ge T$. Thus by system (1) we have

$$Z_i(t) < -\frac{\alpha}{2}\sum_{j=1}^{n}\bigvee_{s=c_{ij}}^{d_{ij}} \beta_{ij}(t,s), \quad t \ge T.$$

Hence from condition (16), we get

$$\lim_{t\to\infty} Z_i(t) = -\infty,$$

this contradiction to $K_i > 0$. So $\lim_{t\to\infty} x_i(t) = 0$, that is $\lim_{t\to\infty} \|x(t)\| = 0$. Thus the proof is completed. □

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一类具分布型滞量的中立型微分系统的稳定性

包 俊 东[1], 胡 永 珍[1], 敖 登[2]

(1. 内蒙古师范大学数学系，呼和浩特 010022; 2. 内蒙古医学院药学系数学组，呼和浩特 010059)

摘 要: 研究了一类具分布型滞量的线性中立型系统，得得到了系统零解的一致稳定性及渐近稳定性的充分条件.

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朱 诚[1]，陈景杰[2]，杨丽君[3]

（1.大连理工大学 图书馆国际期刊咨询室，辽宁 大连 116024；2.大连海事大学 学报编辑部，辽宁 大连 116026；3.沈阳建筑工程学院 学报编辑部，辽宁 沈阳 110015）

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