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A Note on Williams' Algorithm for Interpolating Rationals*

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J. Williams posed a problem, that is, the Chebyshev approximation of oscillating decay-type functions in the form of interpolating rationals on $[0,b]$ (See [3]). The approximated function is $f(x)=B(x)g(x)$, where $B,g\in C[0,b]$, $g(x)>0$, $\forall x\in[0,b]$, and $B(x)$ is the "oscillating" factor satisfying $B(x_r)=0$ for distinct $x_r\in[0,b]$, $r=1,2,\cdots,n+1$. The class of approximating functions is

$$V=\left\{\frac{B(x)}{[L(A,x)]^p}: L(A,x)=\sum_{r=1}^{n+1}a_r\phi_r(x)>0,\quad x\in[0,b]\right\} \tag{1}$$

where $\{\phi_1,\cdots,\phi_{n+1}\}$ forms a Chebyshev set on $[0,b]$ and $p>0$. We hope to obtain the best approximation of $f(x)$ in V that keeps the conditions interpolating at the zeros of $f(x)$. In [4], J. Williams and G. D. Taylor interpreted that the existence of the best approximation can not be insured.

If the best approximation exists, as J. Williams pointed out, it may be calculated by Remes' exchange algorithm, and that the system of equations

$$|B(x_r^{(k)})|\{g(x_r^{(k)})-[L(A_k,x_r^{(k)})]^{-p}\}=(-1)^r\lambda_k,\quad r=1,2,\cdots,n+1 \tag{2}$$

has unique solution (A_k,λ_k) leading to $\frac{B(x)}{[L(A_k,x)]^p}$, if $B(x_r^{(k)})\neq 0$, $r=1,2,\cdots,n+1$. [3, theorem 4.1]

In [1], C. B. Dunham deemed that claim on solvability of (2) is incorrect. His approach to the proof is as follows: Let $L(C,x)$ change signs on $[0,b]$ and be positive on the subinterval I on which $B(x)$ is nonzero. Let $x_1^{(k)},\ldots,x_{n+1}^{(k)}$ be distinct points in I. Choose g such that $g(x_r^{(k)})=[L(C,x_r^{(k)})]^{-p}$, $r=1,2,\cdots,n+1$, then $A_k=C$, $\lambda_k=0$ are unique solution to (2). But $[L(C,x)]^{-p}$ has some poles on $[0,b]$, or $[L(C,x)]^{-p}\cdot B(x)\notin V$.

Now we should point out: if we obtain generally the starting points $X_1:\{x_1^{(1)}\ldots,x_{n+1}^{(1)}\}$ through a starting approximation $B(x)[LA_0,x)]^{-p}$ as has been interpreted in [2] (but not necessarily close sufficiently to the system of alternate points of the

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best approximation), then C. B. Dunham' demonstration would be impossible to find a footing. For if this were not true or, in other words, C. B. Dunham' situation arose, according to the appointment of Remes algorithm the system of the points $X_k: \{x_1^{(k)}, \dots, x_{n+1}^{(k)}\}$ would possess the following characters,

1° $g(x)-[L(A_{k-1}, x)]^{-p}$ alternates signs on X_k;

2° The point τ at which $|B(x)||g(x)-[L(A_{k-1}, x)]^{-p}|$ reaches his maximum on $[0,b]$ is contained in X_k.

From 1° we have

$$g(x_r^{(k)})-[L(A_{k-1}, x_r^{(k)})]^{-p}=[L(C, x_r^{(k)})]^{-p}-[L(A_{k-1}, x_r^{(k)})]^{-p}=(-1)^r u_r$$
$$r=1,2,\cdots,n+1 \tag{3}$$

where $u_r>0$. (3) shows $L(C, x)-L(A_{k-1}, x)$ has at least n zeros on $[0, b]$. By the characters of Chebyshev system this implies

$$L(C, x)\equiv L(A_{k-1}, x) \tag{4}$$

Hence

$$g(x_r^{(k)})-[L(A_{k-1}, x_r^{(k)})]^{-p}=[L(C, x_r^{(k)})]^{-p}-[L(A_{k-1}, x_r^{(k)})]^{-p}=0,$$
$$r=1,2,\cdots,n+1. \tag{5}$$

From 2°, (5) implies

$$\max_{x\in[0,b]}|B(x)g(x)-B(x)[L(A_{k-1}, x)]^{-p}|=0. \tag{6}$$

Because $B(x)$ has only the finite zeros on $[0,b]$, then

$$g(x)\equiv[L(A_{k-1}, x)]^{-p} \tag{7}$$

From (4) and (7),

$$g(x)\equiv[L(C, x)]^{-p} \tag{8}$$

But this can not arise out of the hypothesis on g.

References

[1] Dunham, C. B., A difficulty in Williams' algorithm for interpolating rationals, *Math. Comput.*, **29** (1975) 552—553.

[2] Ralston, A. and Wilf, H. S., Mathmatical methods for digital computers, Vol. II, John Wiley & Sons. Inc. 1968.

[3] Williams, J., Numerical Chebyshev approximation by interpolating rationals, *Math. Comput.*, **26** (1972), 199—206.

[4] Williams, J. and Taylor, G. D., Existence questions for the problem of Chebyshev approximation by interpolating rationals, *Math. Comput.*, **28** (1974), 1097—1103.

关于 Williams 的具有插值条件的有理切比雪夫逼近问题的一个注记

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提 要

J. Williams 在[3]中提出了对振荡衰减函数的具有插值条件的有理切比雪夫逼近问题。C. B. Dunham 于 1975 年在[1]中指出了 Williams 的算法中在理论上存在一个漏洞。本文指出，如[2]中所说，则[1]中 C. B. Dunham 的论证事实上不可能发生。