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# ON A CONJECTURE ABOUT NONLINEAR FILTER FOR BILINEAR SYSTEMS*

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## Abstract

Ln this note we prove that it is not possible to get a better linear filter for bilinear systems by using the updated innovation.

we consider the bilinear systems described by

$$\begin{cases} dx_t = F_t x_t dt + G_t x_t dw_t, & 0 \leqslant t \leqslant T, \\ dy_t = H_t x_t dt + M_t x_t dv_t, & 0 \leqslant t \leqslant T. \end{cases} \tag{1}$$

Here, for simplicity, $x$ and $y$ are one dimensional stochastic processes. We assume shat $w$ and $v$ are standard Brownian motions independent of $x_0$, which is a zero mean random variable; the process noise $w$ is independent of the observation noise $v$, and the time functions $F_t$、$G_t$、$H_t$ and $M_t$ are given.

It is well known[1] that the best linear filter of the systems (1) and (2) is given by

$$d\hat{x}_t = F_t \hat{x}_t dt + K(t) d\nu_t, \tag{2}$$

$$K(t) = P(t) H_t \Sigma_\nu^{-1}(t), \tag{3}$$

$$d\nu_t = dy_t - H_t \hat{x}_t dt, \tag{4}$$

$$\Sigma_\nu(t) = M(t) X(t) M(t), \tag{5}$$

$$\dot{X}(t) = F_t X(t) + X(t) F_t + G_t X(t) G_t, \tag{6}$$

$$\begin{aligned} \dot{P}(t) &= F_t P(t) + P(t) F_t + G_t X(t) G_t - K(t) \Sigma_\nu(t) K(t) \\ &= F_t P(t) + P(t) F_t + G_t X(t) G_t - P(t) H_t \Sigma_\nu^{-1}(t) H_t P(t). \end{aligned} \tag{7}$$

Now, a natural approach to improving on the linear estimate is to follow the linear filter by a postprocessor, as shown in Figure 1. If the criterion is least square mean error, the estimate $\hat{\hat{x}}_t$ generated by the postprocessor is the conditional mean $E[x_t | \hat{x}_t]$.

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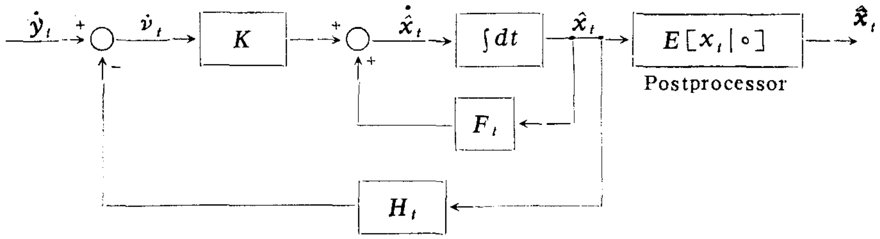


Figure 1. Optimal linear filter with nonlinear postprocessor

An idea that seems reasonable would be to update the innovation process $d\nu_t$ by using the post-processor's estimate $\hat{\hat{x}}_t$ instead of the linear estimate $\hat{x}_t$. Then we use the update innovation to produce a suboptimal filter of the form as shown in Figure 2.

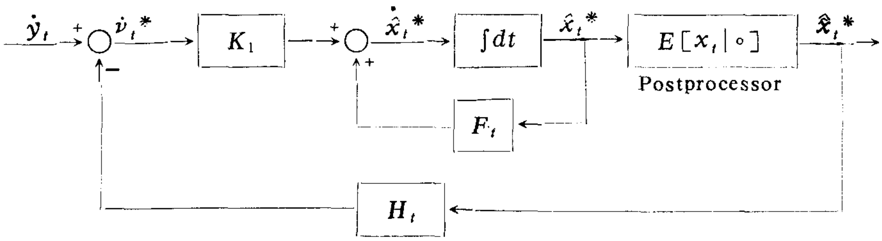


Figure 2. Nonlinear filter using updated innovations.

It is easy to write the filter equation for the system seen in Figure 2. In fact, we have the following equations for the filter:

$$d\hat{x}_t^* = F_t\hat{x}_t^* dt + K_1(t)d\nu_t^*, \tag{8}$$

$$K_1(t) = P_1(t)H_t\Sigma_{\nu_t}^{-1}(t), \tag{9}$$

$$d\nu_t^* = dy_t - H_t\hat{\hat{x}}_t dt, \tag{10}$$

$$\dot{P}^*(t) = F_tP^*(t) + P^*(t)F_t + G_tX(t)G_t - P_1(t)H_t\Sigma_\nu^{-1}(t)H_tP_1(t), \tag{11}$$

where

$$P_1(t) \triangleq E[(x_t - \hat{\hat{x}}_t^*)^2] = P^*(t) - E[\hat{x}_t^* - \hat{\hat{x}}_t^*)^2]. \tag{12}$$

Thus, we propose the problem -Is the estimate $\hat{\hat{x}}_t^*$ better than the estimate $\hat{\hat{x}}_t$. Although this conjecture seems quite reasonable, we prove that it is not possible to get a better linear filter by using the updated innovations, i. e. the estimate $\hat{x}_t^*$ is not better than the estimate $\hat{x}_t$.

Our purpose is to prove that $P^*(t) > P(t)$. Noticing that if $P^*(t)$ satisfies the equation

$$\dot{P}^*(t) = F_tP^*(t) + P^*(t)F_t + G_tX(t)G_t - P^*(t)H_t\Sigma_\nu^{-1}(t)H_tP^*(t),$$

then the $P^*(t)$ is equal to $P(t)$, because their initial conditions are equal.

Now, from (12) we have

$$P_1(t)H_t\Sigma_\nu^{-1}(t)H_tP_1(t) - P^*(t)H_t\Sigma_\nu^{-1}(t)H_tP^*(t) > 0. \tag{13}$$

Thus we conclude that [2]

$$p^*(t) - p(t) \geqslant 0 \tag{14}$$

Therefore. using the updated innovation cannot improve the linear estimate. The reason can be explained by the fact that in deriving the best linear filter we use the "true" innovation, but the updated innovation by using postprocessor is not the "true" innovation.

## References

[1] Vallot, L. C., "Filtering for Bilinear Systems," M. S. thesis, Massachusetts Institute of Technology, 1981.

[2] Reid, W. T., "Riccati Differential Equations", New York, Academic Press, 1972.