Numbers of Topologies and Lattices of Sets on an Infinite Set*

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Let X be an infinite set, $K=\{\tau:\tau$ is a topology on $X\}$, define $\tau\sim\sigma$ iff $(\exists f)$ (f is a homeomorphism from (X,τ) to (X,σ)), $\tau\cong\sigma$ iff $(\exists f)$ (f is a lattice-isomorphism from τ to σ), $\langle\tau\rangle=\{\sigma:\sigma\sim\tau\}$, $\widehat{\tau}=\{\sigma:\sigma\cong\tau\}$ (for a given $\tau\in K$), $K^*(\subset K)$ is an incomparable class iff $(\forall\tau\in K^*)(\forall\sigma\in K^*)(\tau\neq\sigma\rightarrow\tau$ and σ are incomparable).

Theorem 1 If $K_2=\{\langle\tau\rangle:\tau\in K$ & (X,τ) is a Hausdorff space$\}$, then $|K_2|=2^{2^{|X|}}=\exp(\exp(|X|))$.

Theorem 2 If $C_2=\{\tau:\tau\in K$ & (X,τ) is a Hausdorff space$\}$, for an incomparable class C_2^* $(\subset C_2)$ we define $K_2^*=\{\langle\tau\rangle:\tau\in C_2^*\}$, then $\max\{|K_2^*|\}=\sup\{|K_2^*|\}=|K|=\exp(\exp(|X|))=2^{2^{|X|}}$.

Theorem 3 If $K_L=\{\widehat{\tau}:\tau\in K\}$, then $|K_L|=\exp(\exp(|X|))$.

Theorem 4 If $K_C=\{\tau:\tau\in K$ & τ is a complete lattice of sets with respect to the operations of arbitrary intersection and union$\}$, then $|K_C|=2^{|X|}=\exp(|X|)$.

Theorem 5 If $K_{CT}=\{\langle\tau\rangle:\tau\in K_C\}$, then $|K_{CT}|=|K_C|=2^{|X|}=\exp(|X|)$.

Theorem 6 If $K_{CL}=\{\widehat{\tau}:\tau\in K_C\}$, then $|K_{CL}|=\exp(|X|)$.

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