

The Largest Cardinals of Isomorphic Class of Topology-set-lattices on an Infinite Set *

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Let X be an infinite set, $K = \{\tau: \tau \text{ is a topology on } X\}$, define $\tau \cong \sigma$ iff $(\exists f)$ $\langle f \text{ is a lattice-isomorphism from } \tau \text{ to } \sigma \rangle$, for a given $\tau \in K$ we define $\hat{\tau} = \{\sigma: \sigma \in K \text{ \& } \sigma \cong \tau\}$, $K^* (\subset K)$ is an incomparable class iff $(\forall \tau \in K^*) (\forall \sigma \in K^*) (\tau \not\cong \sigma \rightarrow \tau \text{ and } \sigma \text{ are incomparable})$, $M = \{K^*: K^* \text{ is an incomparable class}\}$.

Theorem 1 $\sup \{|\hat{\tau}|: \tau \in K\} = \max \{|\hat{\tau}|: \tau \in K\} = |K| = 2^{2^{|X|}} = \exp(\exp(|X|))$.

Theorem 2 $\sup \{\sup \{|\hat{\tau}|: \tau \in K^*\}: K^* \subset K \text{ \& } K^* \text{ is an incomparable class}\} = \sup \{|\hat{\tau}|: \tau \in K^* \text{ \& } K^* \in M\} = \max \{|\hat{\tau}|: \tau \in K^* \text{ \& } K^* \in M\} = 2^{2^{|X|}} = \exp(\exp(|X|)) = \sup \{|\hat{\tau}|: \tau \in K\} = \max \{|\hat{\tau}|: \tau \in K\} = |K|$.

In the proof of the above-mentioned theorems, the AC and GCH will be used.

References

- [1] Yang An-zhou and Zheng Chong-you, Numbers of Topologies and Lattices of Sets on an Infinite Set, J. M. R. E., Vol. 4, 1(1984), 146.
- [2] Zheng Chong-you and Yang An-zhou, The Largest Cardinals of Homeomorphic Class and Isomorphic Class of Set-lattices on an Infinite Set, J.M.R.E., Vol. 4, 1 (1984), 94.

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