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Some Results on M-Hyponormal Operators*

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Let $\mathscr{B}(\mathscr{H})$ denote the algebra of all bounded linear operators on a Hilbert space $\mathscr{H}$. $T\in\mathscr{B}(\mathscr{H})$ is called a dominant operator if for each $\lambda\in\mathbf{C}$ there exists a number $M_\lambda>0$ such that $\|(T^*-\bar\lambda)x\|\leqslant M_\lambda\|(T-\lambda)x\|$ for all $x\in\mathscr{H}$. Furthermore, if the constants M_λ are bounded by a positive number M, then T is called a M-hyponormal operator. In this paper, we give some properties of the operators which are dominant or M-hyponormal, and introduce a class $(M\text{-}G)$ of operators which are M-hyponormal satisfying some growth condition.

1. Some Properties of Dominant and M-hyponormal Operators.

Proposition 1.1 If T is dominant, then

(1) $0\in\sigma([T^*, T])$;

(2) $\sigma(T)\cap\mathbf{R}\neq\phi$ implies $0\in\sigma(\mathrm{Im}\,T)$, and $\sigma(T)\cap i\mathbf{R}\neq\phi$ implies $0\in\sigma(\mathrm{Re}\,T)$;

(3) $\mathrm{Re}\,\sigma(T)\subset\sigma(\mathrm{Re}\,T)$ and $\mathrm{Im}\,\sigma(T)\subset\sigma(\mathrm{Im}\,T)$.

Proposition 1.2 If T is an invertible dominant operator, then T^{-1} is also. If T is invertible and M-hyponormal, then T^{-1} is M_1-hyponormal with $M_1=\|T\|\cdot\|T^{-1}\|\cdot M$.

Proposition 1.3 (1) The set of M-hyponormal operators in $\mathscr{B}(\mathscr{H})$ is norm-closed;

(2) If T is M-hyponormal and N is normal such that $TN=NT$, then both $T+N$ and NT are M-hyponormal, too.

Proposition 1.4 Let T be a dominant operator. If $\lambda_0\in\sigma(T)$ such that the descent of $T\text{-}\lambda_0$ is finite, then λ_0 is an isolated eigenvalue of T.

Proposition 1.5 Let T be a dominant operator and $f(\lambda)$ an analytic function which is defined on some open set $\Omega\supset\sigma(T)$ and is not a constant in any component of Ω. If $f(T)$ is normal, then T is normal.

Proposition 1.6 If T is M-hyponormal and satisfying one of the following conditions, then T must be normal.

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(1) $\sigma(T^*)$ has analytic capacity zero;

(2) The continuous capacity of $\sigma(T)$ and the analytic capacity of $[\pi_0(T^*)\cap\partial\sigma(T^*)]^-$ are zero;

(3) $\sigma(T)$ is countable;

(4) T is polynomially compact.

Corollary 1.1 If T is M-hyponormal and λ_0 is an isolated point in $\sigma(T)$, then $\lambda_0\in\pi_0(T)$.

Proposition 1.7 Let T be a M-hyponormal operator. If Im T is compact and the restriction of T to $[\bigvee_{\lambda\in\pi_0(T)}F_\lambda]^\perp$ has no residual spectrum, where $F_\lambda=\{x\in\mathscr{H};\ (T-\lambda)x=0\}$, then T is normal.

2. The Weyl's Theorem for M-hyponormal operators.

For $T\in\mathscr{B}(\mathscr{H})$, the Weyl spectrum of T is the set $w(T)=\bigcap_{K\in\mathscr{K}(\mathscr{H})}\sigma(T+K)$, where $\mathscr{K}(\mathscr{H})$ denotes the set of all compact operators in $\mathscr{B}(\mathscr{H})$. We say Weyl's theorem holds for T if $w(T)$ consists precisely of all points in $\sigma(T)$ except the isolated eigenvalues of finite multiplicity.

Theorem 2.1 Weyl's theorem holds for any M-hyponormal operator.

3. The Operators of class $(M\text{-}G)$.

Definition We say an operator T belongs to the class $(M\text{-}G)$ if T is M-hyponormal and there exists a non-negative function $g_T(t)$ on $(0, +\infty)$ satisfying following two conditions:

(1) $\overline{\lim\limits_{n\to\infty}}[g_T(n)]^{1/n}<2$;

(2) $\|(T-\lambda)^n x\|^2\leqslant M^{\varphi_T(n)}\|(T-\lambda)^{2n}x\|\cdot\|x\|$, for all $\lambda\in\mathbf{C}$, $x\in\mathscr{H}$ and $n=1,2,\cdots$, where $\varphi_T(n)=g_T(\log n/\log 2)$.

Proposition 3.1 Let $T\in(M\text{-}G)$ and N be a normal operator commuting with T, then both $T+N$ and NT are of class $(M\text{-}G)$.

Theorem 3.2 If $T\in$ (M-G), then for some constant $\alpha(T)>0$, $r\sigma(T)\geqslant\|T\|/M^{\alpha(T)}$, where $r\sigma(T)$ denotes the spectral radius of T.

Theorem 3.3 If $T\in(M\text{-}G)$, then T satisfies the condition (G_1), i. e., there exists a constant $\alpha(T)$ such that $\|(T-\lambda)^{-1}\|\leqslant\alpha(T)/\text{dist}(\lambda, \sigma(T))$ for all $\lambda\overline{\in}\sigma(T)$.

Corollary 3.1 Suppose $T\in(M\text{-}G)$ and $\sigma(T)$ is contained in a rectifiable curve, then T is normal.

Corollary 3.2 Suppose $T\in(M\text{-}G)$. If $\sigma(\text{Im } T)$ is countable and has at most finitely many accumulation points, then T is normal.

Corollary 3.4 If $T\in(M\text{-}G)$ and Im T is polynomially compact, then T is normal.

Theorem 3.4 Every $T \in (M\text{-}G)$ satisfies a local growth condition of order 1, i.e. for every closed set $\delta \subset \mathbf{C}$ and every $x \in X_T(\delta)$, there exists an analytic function $f: \mathbf{C} \setminus \delta \to \mathscr{H}$ such that $(T-\lambda) f(\lambda) \equiv x$ and $\|f(\lambda)\| \leqslant a(T)[\operatorname{dist}(\lambda, \delta)]^{-1}$, where $a(T)$ is a positive number independent of δ and x.

Theorem 3.5 If $T \in (M\text{-}G)$, then T satisfies Dunford's condition (C), that's $X_T(\delta)$ is closed for every closed set $\delta \subset \mathbf{C}$.

Corollary 3.5 Let $T \in (M_1 - G)$ and $S \in (M_2 - G)$. If there exist quasiaffinities X and Y such that $XT = SX$ and $TY = YS$, then $\sigma(T) = \sigma(S)$, If either X or Y is compact, then we also have $\sigma_e(T) = \sigma_e(S)$.

Corollary 3.6 Suppose $T \in (M\text{-}G)$. If there is a nonzero $x \in \mathscr{H}$ such that $\|T^n x\| \leqslant K t^n$ $(n = 1, 2, \cdots)$, where K、t are constants and $0 < t < \|T\| / M^{a(T)}$ ($a(T)$ is same as in Th. 3.2), then T has a nontrivial invariant subspace.

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