Note on the Concept of Microscopic Measure in *R*

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As is known [1], $\mathbb{N}=\{\nu\mid 1\leqslant\nu<(\omega)\}$ is the non-Cantorian model of natural numbers in which (ω) denotes the galaxy of the infinite integer ω, and $\nu<(\omega)$ means that $\nu<\omega\pm n$ $\forall n\in N$ (set of ordinary natural numbers). All the infinite integers contained in $\mathbb{N}$ constitute the flying segment and may be denoted by $\widetilde{N}$. It may be viewed that $\mathbb{N}$ is the telescopic contents of N when N is embedded and extended in *R. Correspondingly every bounded sequence of points $\{x_n\mid n\in N\}\subset R$ has its microscopic contents $\{x_\nu\mid\nu\in\mathbb{N}\}\subset{}^*R$, where $\{x_\nu\}$ is the natural extension of $\{x_n\}$ (cf. Axiom 5 of Keisler's Calculus).

Given a bounded infinite set S in R. The microscopic contents $^*S$ of S is defined to be the set consisting of all the microscopic contents of the sequences contained in S. Moreover, the set consisting of all the standard parts of the elements of $E\subset{}^*R$ may be denoted by st(E).

Definition. For any bounded infinite set $S\subset R$ with its microscopic contents $^*S\subset{}^*R$, the Lebesgue measure of the standard set st$(^*S)$ is called the microscopic (outer) measure of S (embedded in $^*R$), and may be written as $^*m(S)=\text{mes}(\text{st}(^*S))=\text{mes}(\bar{S})$, where $\bar{S}$ is the closure of S.

Example. Any set $E\subset R$ dense in the interval (a,b) has the microscopic outer measure $^*m(E)=b-a$ when it is embedded in $^*R$. In particular, for the set Q of rational points embedded in $^*R$ we have $^*m(Q\cap(a,b))=b-a$.

Clearly the present situation is entirely different from and much simpler than that in the ordinary Lebesgue measure theory. Moreover, it may be worth noting that an equivalent formulation of the microscopic measure concept can be accomplished by using monads as covering sets for points in $^*R\setminus R$.

Reference

Xu Lizhi, Selected Topics on the Methodology of Mathematics, (Chinese), HUST Press, Wuhan, China, 1983, chapter 7, §3-§5.

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