第4卷 第4期
1984年10月

数学研究与评论
JOURNAL OF MATHEMATICAL
RESEARCH AND EXPOSITION

Vol.4 No.4
Oct. 1984

Some Topological Properties of Hyperspace*

Yang Xiu (杨旭)

(Si-Ping Teacher's College)

Certain topological properties of the hyperspaces have been studied in [1],[2] and[3]. In this paper we will discuss some topological properties of hyperspace $(\mathscr{B}, 2^T)$, and will generalize related results of Z. Michael [1], A. Frank, Chimenti, [2], 寿纪麟[4].

Let (X,T) is a topological space,

$$\mathscr{A}(X)=\{Z\subset X: Z\neq\emptyset\} \qquad 2^X=\{Z\in\mathscr{A}(X): Z=\bar{Z}\}$$

$$\mathscr{K}(X)=\{Z\in\mathscr{A}(X): Z \textit{ is compact}\} \qquad \mathscr{F}(X)=\{Z\in\mathscr{A}(X): |Z|<\aleph_0\}$$

$$\langle U_1,\cdots,U_n\rangle=\left\{Z\in\mathscr{B}: Z\subset\bigcup_{i=1}^{n}U_1,\ Z\cap U_i\neq\emptyset, i=1,\cdots,n\right\},\ \text{其中}\ \mathscr{B}\subset\mathscr{A}(X).$$

2^T is the vietoris topology in $\mathscr{B}$. Our main results are as follows:

Theorem 1 Let A is a closed subset in X, then:

$\{Z: Z\subset A\}$ is a closed subset in $(\mathscr{B}, 2^T)$.

$\{Z: Z\cap A\neq\emptyset\}$ is a closed subset in $(\mathscr{B}, 2^T)$.

Theorem 2 $\mathrm{Cl}\langle V_1,\cdots,V_n\rangle=\langle\bar{V}_1,\cdots,\bar{V}_n\rangle$, where "Cl" is the closure operator in $(\mathscr{B}, 2^T)$, and $V_i\subset X$, $i=1,2,\cdots n$.

lemma $\mathscr{B}\supset\mathscr{F}(X)\Leftrightarrow$ *if* $\langle U_1,\cdots U_n\rangle\subset\langle V_1,\cdots,V_m\rangle$, then $\bigcup_{i=1}^{n}U_i\subset\bigcup_{j=1}^{m}V_j$, and $\forall\ V_j$, $\exists\ U_i$ so that $U_i\subset V_j$, $U_i,V_j\subset X$.

Theorem 3 If $\mathscr{B}\supset\mathscr{F}(X)$, then $(\mathscr{B}, 2^T)$ is separable if and only if (X,T) is separable.

Theorem 4 If $\mathscr{B}\supset\mathscr{K}(X)$, then $(\mathscr{B}, 2^T)$ is compact if and only if (X,T) is compact.

Theorem 5 Let (X,T) is a T_1 topological space, if $\mathscr{B}\supset 2^X$, then $(\mathscr{B}, 2^T)$ is compact if and only if (X,T) is compact.

Theorem 6 If $\mathscr{B}\supset 2^X\cap\mathscr{K}(X)$, and $\forall\ x\in X$, $\{x\}\in\mathscr{B}$, then $(\mathscr{B}, 2^T)$ is compact if and only if (X,T) is compact.

The author expresses his warmest gratitude to Prof. Fang Jialin for suggestions.

References

[1] Michael, E, Topologies on spaces of subsets, *Trans. Amer. Math.* Soc. 71 (1951), 152—182.
[2] Frank, A,. Chimenti,Tychonoff's theorem hyperspace, *Proc. Amer, Math. Soc.* Vol.37, N1(1973).
[3] Попов, В, В, О пространстве замкнутых подмножеств ДАН, СССР, 1976 том 229 M5.
[4] 寿纪麟，超空间上的一些拓扑关系。数学研究与评论，1983年第3卷第1期，93—94.
[5] 杨旭，关于超空间的几个基数函数，四平师范学院学报，1982年第二期20—28.

* Received May 27, 1983.