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The Normal Form on the Inverses of M-Matrices*

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Let $\mathcal{M}$ denote the set of all $n\times n$ nonsingular M-matrices. If $A\in\mathcal{M}$, then there exists a permutation matrix P such that

$$PAP^T=B=\begin{pmatrix}B_{11}\cdots B_{1m}\\ \ddots\\ B_{mm}\end{pmatrix},\tag{1}$$

where B_{ii} $(1\leqslant i\leqslant m)$ is an irreducble nonsingular M-matrix, $B_{ij}\leqslant 0$ $(i<j)$, $B_{ij}=0$ $(i>j)$. Let $V=B^{-1}$. Then V is a block upper triangular matrix of the form

$$V=\begin{pmatrix}V_{11}\cdots V_{1m}\\ \ddots\\ V_{mm}\end{pmatrix},\tag{2}$$

where $V_{ii}=B_{ii}^{-1}>0(1\leqslant i\leqslant m)$, $V_{ij}\geqslant 0$ $(i<j)$, $V_{ij}=0$ $(i>j)$. We shall say that (2) is the normal form on the inverse of M-matrix A. The purpose of this note is to reveal some properties of the normal form (2) and the relation between (1) and (2). Thereby, we get the so called null block pattern power invariant property of the normal form (2). (see corollary 1),

Theorem 1 Suppose $B\in\mathcal{M}$, $V=B^{-1}$ anh B, V are block upper triangular matrices of the form (1) and (2) respectively, then

(a) If, for some sequence $i=i_0<i_1<\cdots<i_s<j$, $s\geqslant 0$, each of B_{ii_1}, $B_{i_1i_2}$, $\cdots$, B_{i_sj} is not null matrix, then $V_{ij}>0$.

(b) If $V_{ij}\neq 0$, then there exists at least one sequence $i=i_0<i_1<\cdots<i_s<j$, $s\geqslant 0$, such that each of $B_{ii_1},B_{i_1i_2},\cdots,B_{i_sj}$ is not null matrix.

Theorem 2 Suppose $A\in\mathcal{M}$ and $U=A^{-1}$. Then there exists a permutation P such that $PUP^{-1}=V$ has a form as (2). Furthermore,

(a) $V_{ii}>0$ $(i=1,2,\cdots,m)$, either $V_{ij}>0$ or $V_{ij}=0$ $(i\neq j)$.

(b) If, for some sequence $i<i_1<\cdots<i_s<j$, each of V_{ii_1}, $V_{i_1i_2}$, $\cdots$, V_{i_sj} is not null matrix, then $V_{ij}>0$.

Corollary 1 Suppose $A\in\mathcal{M}$ and $U=A^{-1}$, for arbitrary positive integal q, there exists a permutation matrix P such that $PUP^T=V$ ant $PU^qP^T=V^q$ have same forms as (2). Furthermore, $V_{ij}>0$ if and only if $V_{ij}^q>0$. (V_{ij}^q denote the (i,j) block submatrix of V^q).

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