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Uniform Non-squareness and Flatness in Orlicz Spaces*

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In 1964, R. C. James introduced the concept of uniform non-squareness in normed linear space X: X is said to be locally uniformly non-square, if for each $x\in X$, there exists $l>1$, such that, for any $y\in X$, satisfying

$$\max\{\|x+y\|, \|x-y\|\}\geqslant l\min\{\|x\|, \|y\|\}, \tag{1}$$

X is said to be uniformly non-square, if there exists $l>1$, satisfying (1) for any $x,y\in X$[1]. For Non-squareness there is a closed relation with convexity, flatness, and reflexivity etc.

In 1966, K. Sundaresan[2] discussed uniform non-squareness in Orlicz space $L_M^*(G)$ with respect ot Luxemburg norm $\|\cdot\|_{(M)}$.

In 1981, A. J. Pach and others[3] discussed flatness in Orlicz space $L_M^*(G)$ with respect to Luxemburg norm $\|\cdot\|_{(M)}$.

So far, we haven't seen the discussion about uniform non-squareness and flatness in Orlicz spaces $L_M^*(G)$ with respect to Orlicz norm $\|\cdot\|_{(M)}$.

In this paper, we discuss this problem and obtain some related results.

Theorem 1 Every Orlicz space $L_M^*(G)$ with respect to Orlicz norm $\|\cdot\|_{(M)}$ is locally uniformly non-square.

Corollary No Orlicz space $L_M^*(G)$ with respect to Orlicz norm $\|\cdot\|_{(M)}$ is flat space,

Theorem 2 Orlicz space $L_M^*(G)$ with respect to Orlicz norm $\|\cdot\|_{(M)}$ is uniformly non-square iff

(i) $M(u)$ satisfies Δ_2-condition for largeu.

(ii) there exists u_0, $\delta>0$, such that

$$M(2u)\geqslant(2+\delta)M(u), \qquad u\geqslant u_0.$$

Corollary If $L_M^*(G)$ is uniformly non-square with respect to $\|\cdot\|_{(M)}$, then $L_M^*(G)$ is reflexive.

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