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Integrals of Cauchy Type in C^n Space*

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The earliest paper to study boundary behavior of the integrals of the Cauchy-Martinelli type was [1]. But up to the present, not many boundary behaviors for the integrals of the Cauchy-Fantappié type are discussed. In this paper, we study the boundary behavior for the following form

$$F(Z)=\int_{\partial D} f(\xi)K(\xi,Z) \tag{1}$$

It's a kind of integral with Cauchy-Fantappié kernel more general than that in paper [2], where ∂D, the boundary of a bounded domain D is a smooth hypersurface, E denotes a neighborhood of $\overline{D}$, $f(\xi)$ satisfies Hölder condition with index $\alpha(0<\alpha<1)$ with respect to ξ on ∂D,

$$K(\xi,Z)=\frac{(n-1)!}{(2\pi i)^n}\frac{\omega^*[N(\xi,Z)]}{[M(\xi,Z)]^n}\wedge\omega(\xi),\quad \omega(\xi)=d\xi_1\wedge d\xi_2\wedge\cdots\wedge d\xi_n,$$

$$\omega^*[N(\xi,Z)]=\sum_{j=1}^{n}(-1)^{j-1}N_j dN_1\wedge dN_2\wedge\cdots dN_{j-1}\wedge\cdots dN_{j+1}\wedge\cdots dN_n,$$

$$M(\xi,Z)=\sum_{j=1}^{n}(\xi_j-Z_j)N_j(\xi,Z),\quad N_j(\xi,Z)=(\overline{\xi}_j,\overline{Z}_j)\varphi(\xi,Z)+\theta_j(\xi,Z).$$

Where $N_j(\xi,Z)$ and $\varphi(\xi,Z)$ are functions of class C^2 on $E\times E$. E is a zero free region of $\varphi(\xi,Z)$. $M(\xi,Z)\neq O$, when $\xi\neq Z$. Moreover, $|\theta_j(\xi,Z)|\leqslant O(|\xi-Z|^\beta)$.

$$\left|\frac{\partial\theta_j(\xi,Z)}{\partial\overline{\xi}_k}\right|\leqslant O(|\xi-Z|^r),\ k,\ j=1,2,\cdots,n.\ \beta>1,\ r>0;$$

$$\Omega=\partial D-\partial D\cap S_\varepsilon(t),\ S_\varepsilon(t)=\{\xi\,|\,|\xi-t|^2<\varepsilon^2\}.$$

We obtain the following main results:

Definition If $t\in\partial D$ and if $\lim\limits_{\varepsilon\to 0}\int_\Omega f(\xi)K(\xi,t)$ exists, then $F(t)=\int_{\partial D} f(\xi)K(\xi,t)$ called the Cauchy principal value of (1).

Lemma 1 (i) If $\xi\neq Z$ on E, then

$$\int_{\partial D}K(\xi,Z)=\begin{cases}0, & Z\in E-\overline{D},\\ 1/2, & Z\in\partial D,\\ 1, & Z\in D.\end{cases}$$

(ii) The Cauchy principal value of $\int_{\partial D} f(\xi)K(\xi,Z)$ exists.

(iii) $\Phi(Z)=\int_{\partial D}[f(\xi)-f(t)]K(\xi,Z)$ is a continuous function at the

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point $Z=t$ for all $t\in\partial D$, *i.e.* $\lim\limits_{Z\to t}\Phi(Z)=\Phi(t)$.

Theorem 1 (i) The integral (1) possesses the inner and the outer limit values $F_i(t)=\lim\limits_{\substack{Z\to t\\ Z\in D}}\int_{\partial D}f(\xi)K(\xi,Z)$ and $F_e(t)=\lim\limits_{\substack{Z\to t\\ Z\in E-\bar{D}}}\int_{\partial D}f(\xi)K(\xi,t)$.

(ii) The Сохоцкий-plemelj formulas

$F_i(t)=\int_{\partial D}f(\xi)K(\xi,t)+1/2f(t)$, $F_e(t)=\int_{\partial D}f(\xi)K(\xi,t)-1/2f(t)$ are valid.

Corollary Suppose $f(Z)$ is an analytic function on $\bar{D}$, then $F_i(t)=f(t)$.

Lemma 2 (i) The limit functions $F_i(t)$ and $F_e(t)$ satisfy Hölder condition on ∂D.

(ii) If $t\in\partial D$, then

$$\int_{\partial D}K(\xi,Z)K(t,\xi)=\begin{cases}\dfrac{1}{2}K(t,Z), & Z\in D,\\ -\dfrac{1}{2}K(t,Z), & Z\in E-\bar{D}.\end{cases}$$

Theorem 2 The composite formula

$$\int_{\partial D}K(\tau,t)\int_{\partial D}f(\xi)K(\xi,\tau)=\frac{1}{4}f(t)$$

is valid. If $g(\tau)=2\int_{\partial D}f(\xi)K(\xi,\tau)$, then the reversion formula

$$f(t)=2\int_{\partial D}g(\tau)K(\tau,t)$$

is valid.

Lemma 3 Suppose the kernel $L(\xi,\ \eta)$ satisfies Hölder condition with respect to ξ and η on ∂D, with differential form constructed by complex valued function, then

$$\int_{\partial D}K(\eta,t)\int_{\partial D}f(\xi)L(\xi,\eta)=\int_{\partial D}f(\xi)\int_{\partial D}K(\eta,t)L(\xi,\eta).$$

Theorem 3 If $\varphi(\xi)$ and $L(\xi,t)$ satisfy Hölder condition with respect to ξ and t on ∂D, then the characteristic equation $af+bKf=\varphi$ of the singular integral equation $af+bKf+Lf=\varphi$ has a unique solution $f=\dfrac{a\varphi-bK\varphi}{a^2-b^2}$ in the H classes, and it is equivalent to Fredholm equation $Af+L^*f=\varphi^*$, where $A=a^2-b^2\neq 0$, $L^*=aL-bKL$,

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