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The Asymptotic Stability of Neutral Differential Difference Equation $\frac{d}{dt}(x(t)-Cx(t-\tau))=Ax(t)+Bx(t-\tau)$*

Luo Lai-han (罗来汉)

(Huazhong University of Science and Technology)

The stability of neutral differential difference equation with real coefficients

$$\frac{d}{dt}(x(t)-Cx(t-\tau))=Ax(t)+Bx(t-\tau) \quad (\tau\geqslant 0,\ constant) \tag{1}$$

has been discussed in [1]. It is noted that there exist $\tau(A, B, C)>0$ such that the asymptotic stability of trival solution of (1) and that of

$$\frac{d}{dt}(x(t)-Cx(t))=Ax(t)+Bx(t), \quad C\neq 1 \tag{1'}$$

are equivalent when $0<\tau<\tau(A, B, C)$ in certain conditions. A bound of $\tau(A, B, C)$ has been got from the theorem of N.Tchebotaliof (Н. Чеботарев) and N. Meuman(Н. Мейман)[2].

In this paper, we give necessary and sufficient conditions of the equivalence of the asymptotic stability of trival solution of (1) and (1′).

Lemma The necessary condition of the fact that the roots of the characteristic equation

$$\lambda(1-Ce^{-\lambda\tau})=A+Be^{-\lambda\tau} \tag{2}$$

are all with negative real parts is $|C|<1$[4].

The asymptotic stability of (1) has been discussed when $C=0$[1][3]. Now, we can suppose $C\neq 0$.

It is evident that the real part of the root of (2) is negative when $\tau=0$ if and only if $\frac{A+B}{1-C}<0$. For $\tau>0$, let $\lambda=iy$, (2) becomes

$$iy(1-Ce^{-iy\tau})=A+Be^{-iy\tau},$$

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namely

$$yC\sin(-y\tau)=A+B\cos(-y\tau), \tag{4}$$

$$y(1-C\cos(-y\tau))=B\sin(-y\tau). \tag{5}$$

By discussing on the solutions of simultaneous equations (4) and (5), we have the following theorem since the roots of (2) depend on τ continuously.

Theorem The trival solution of equation (1) is asymptotically stable if and only if

(Ⅰ) The coefficients of the equation(1) satisfy

$$A+B<0,\ A-B\leqslant 0,\ |C|<1$$

or (Ⅱ) The coefficients of the equation(1) satisfy

$$A+B<0,\ A-B>0,\ |C|<1 \text{ and}$$

$$0\leqslant\tau<\sqrt{\frac{1-C^2}{B^2-A^2}}\cos^{-1}\left(-\frac{A-BC}{B-AC}\right)\quad(0<\cos^{-1}x<\pi).$$

A matter worthy of note is that [3] is a special case of the above theorem ior $C=0$.

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