第5卷 第1期
1985年1月

数学研究与评论
JOURNAL OF MATHEMATICAL RESEARCH AND EXPOSITION

Vol.5 No.1
Jan. 1985

Finite Difference Method of Boundary Problem for Nonlinear Pseudo-Hyperbolic Systems*

Zhou Yu-lin(周毓麟)

(Post box No. 8009, Beijing)

§1 In the study of the practical problems, such as the forced vibration of plane boundary layer, the transfer of the bioelectric signal in aminal nervous systems and so forth, the linear and nonlinear equations with the main part of form $u_{tt}-u_{xxt}$ of pseudo-hyperbolic type often appear. Many authors have paid attention to solve various problems for the linear and nonlinear pseudo-hyperbolic equatons[1-6]. For a fairly general family of nonlinear pseudo-hyperbolic systems, which contains above mentioned equations as simple special cases, the global solutions of the periodic boundary problems and the initial problems are obtained by Galerkin's method in[7].

In [8] the nonlinear pseudo-hyperbolic system

$$(-1)^M u_{tt}+A(x,t,u,u_x\cdots,u_{x^{2M-1}},u_t,u_{xt},\cdots,u_{x^{M-1}t})u_{x^{2M}t}$$
$$=F(x,t,u,u_x,\cdots,u_{x^{2M-1}},u_t,u_{xt},\cdots,u_{x^{M-1}t}) \tag{1}$$

with the boundary conditions

$$u_{x^k}(0,t)=u_{x^k}(l,t)=0 \qquad (k=0,1,\cdots,M-1) \tag{2}$$

and the initial conditions

$$u(x,0)=\varphi(x),$$
$$u_t(x,0)=\psi(x). \tag{3}$$

is considered in rectangular domain $Q_T=\{0\leqslant x\leqslant l;0\leqslant t\leqslant T\}$, where $u(x,t)=(u_1(x,t),\cdots,u_m(x,t))$ is a m-dimensional vector valued function.

Assume that the following conditions are fulfilled.

(I) $A\equiv A(x,t,p,q)\equiv A(x,t,p_0,p_1,\cdots,p_{2M-1},q_0,q_1,\cdots,q_{M-1})$ is a m×m positively definite continuous matrix in $E=\{(x,t)\in Q_T;\ p_0,p_1,\cdots,p_{2M-1},q_0,q_1,\cdots,q_{M-1}\in R^m\}$ i. e, for any m-dimensional vector $\xi\in R^m$, $(\xi,A(x,t,p,q)\xi)\geqslant a|\xi|^2$, where $a>0$, $(x,t,p,q)\in E$.

(II) $F\equiv F(x,t,p,q)\equiv F\{x,t,p_0,p_1,\cdots,p_{2M-1};\ q_0,q_1,\cdots,q_{M-1}\}$ is a m-dimensional

*Received Feb. 5, 1983.

vector valued continuous function in E, satisfying the relation

$$|F(x,t,p,q)| \leqslant K_1\{\sum_{k=0}^{2M-1}|p_k| + \sum_{k=0}^{M-1}|q_k| + 1\}, \tag{4}$$

where K_1 is a constant.

(Ⅲ) $\varphi(x)$ and $\psi(x)$ are two m-dimensional vector valued initial functions, belonging to $C^{(2M)}([0,l])$ and $C^{(M)}([0,l])$ respectively and satisfying the boundary conditions (2), i. e., $\varphi(x)$ and $\psi(x)$ with their derivatives up to order $M-1$ vanish at the ends $(x=0,l)$ of the the segment $[0,l]$.

(Ⅳ) $A(x,t,p,q)$ and $F(x,t,p,q)$ are Lipschitz continuous in $E_R=\{(x,t)\in Q_T;$ $|p_k|$, $|q_{k'}|\leqslant R, k=0,1,\cdots,2M-1, k'=0,1,\cdots,M-1\}$ for any $R>0$.

We know that under the conditions (Ⅰ), (Ⅱ), (Ⅲ) and (Ⅳ), the boundary problem(2), (3) for the nonlinear pseudo-hyperbolic system (1) has an unique solution $u(x,t)\in Z\equiv W_2^{(1)}((0,T);W_2^{(2M)}(0,l))\cap W_\infty^{(1)}((0,T);W_2^{(M)}(0,l))\cap W_2^{(2)}((0,T);L_2(0,l))$. The purpose of this note is to study the boundary problem (2), (3) for the system (1) by the finite difference method, under the above conditions (Ⅰ), (Ⅱ), (Ⅲ) and (Ⅳ).

We adopt the notations and conventions of [9].

Let $\{u_j\}$ be a discrete function defined on $x_j=jh, j=0, 1,\cdots,J$, $Jh=l$. Denote the scalar product of two discrete functions $\{u_j\}$ and $\{v_j\}$ by $(u,v)_h=\sum_{j=0}^{J}u_jv_jh$. and $\|u\|_h=(u,u)_h$. Also $\|u\|_\infty=\max_{j=0,1,\cdots,J}|u_j|$.

We take the notation $\Delta_+u_j=u_{j+1}-u_j$, $\Delta_-u_j=u_j-u_{j-1}$. Similarly, we have $\|\delta_hu\|_h^2=\sum_{j=0}^{J-1}\left|\frac{\Delta_+u_j}{h}\right|^2h=\sum_{j=1}^{J}\left|\frac{\Delta_-u_j}{h}\right|^2h$, $\|\delta_hu\|_\infty=\max_{j=0,1,\cdots,J-1}\left|\frac{\Delta_+u_j}{h}\right|$ and so on.

The following lemmas[9] are used in later discussions.

LEMMA 1 For any discrete function $\{u_j\}$ $(j=0,1,\cdots,J)$ on the finite interval $[0,l]$, there are the interpolation formula

$$\|\delta_h^ku\|_h \leqslant K_2\|u\|_h^{1-\frac{k}{n}}\left(\|\delta_h^nu\|_h+\frac{\|u\|_h}{l^n}\right)^{\frac{k}{n}}, \quad k=0,1,\cdots n \tag{5}$$

and

$$\|\delta_h^ku\|_\infty \leqslant K_3\|u\|_h^{1-\frac{k+\frac{1}{2}}{n}}\left(\|\delta_h^nu\|_h+\frac{\|u\|_h}{l^n}\right)^{\frac{k+\frac{1}{2}}{n}}, \quad k=0,1,\cdots,n-1, \tag{6}$$

where K_2 and K_3 are constants independent of h, l and $\{u_j\}$.

LEMMA 2 For any discrete function $\{u_j\}$ $(j=0,1,\cdots,J)$ on the finite interval $[0,1]$ and any given $\varepsilon>0$, there exists a constant $K(\varepsilon,n)$, depending only on ε and n, such that

$$\|\delta_h^ku\|_h \leqslant \varepsilon\|\delta_h^nu\|_h + K(\varepsilon,n)\|u\|_h \tag{7}$$

and

$$\|\delta_h^k u\|_\infty \leqslant \varepsilon \|\delta_h^n u\|_h + K(\varepsilon, n)\|u\|_h, \tag{8}$$

where $k=0, 0, 1\cdots, n-1$.

LEMMA 3 For any discrete function $\{u_j\}$ $(j=0,1,\cdots,J)$, satisfying the homogeneous boundary conditions $u_i = u_{J-i} = 0$ $(i=0, 1,\cdots, M-1)$, there is

$$\|\delta_h^k u\|_h \leqslant K_4 l^{M-k} \|\delta_h^M u\|_h \qquad (k=0,1,\cdots,M-1), \tag{9}$$

where K_4 is independent h, l and $\{u_j\}$.

§2 Suppose that the rectangular domain Q_T is divided into small grids by the parallel lines $x=x_j$ $(j=0,1,\cdots,J)$ and $t=t^n$ $(n=0,1,\cdots,N)$, where $x_j=jh$, $t^n=n\Delta t$ and $Jh=l$, $N\Delta t=T$. Denote the m-dimensional vector valued function on the grid point (x_j,t^n) by v_j^n $(j=0,1,\cdots,J;\ n=0,1,\cdots,N)$.

Let us construct the finite difference scheme

$$(-1)^M \frac{v_j^{n+1} - 2v_j^n + v_j^{n-1}}{\Delta t^2} + \tilde{A}_j^{n+a} \frac{\Delta_+^M \Delta_-^M (v_j^{n+a} - v_j^{n-1+a})}{h^{2M}\Delta t} = F_j^{n+a}$$

$$(j=M,\cdots,J-M;\ n=1,\cdots,N-1) \tag{1}_h$$

corresponding to the nonlinear pseudo-hyperbolic system (1), where

$$\tilde{A}_j^{n+a} = A(x_{j*}, t^{n*}, \overset{*}{p}{}_{0j}^{n+a}, \overset{*}{p}{}_{1j}^{n+a}, \cdots, \overset{*}{p}{}_{3M-1,j}^{n+a}, \overset{*}{q}{}_{01}^{n+a}, \overset{*}{q}{}_{1j}, \cdots, \overset{*}{q}{}_{M-1j}^{n+a}),$$

$$F_j^{n+a} = F(x_{j*}, t^{n*}, \bar{p}_{0j}^{n+a}, \bar{p}_{1j}^{n+a}, \cdots, \bar{p}_{2M-1j}^{n+a}, \bar{q}_{01}^{n+a}, \bar{q}_{1j}^{n+a}, \cdots, \bar{q}_{M-1j}^{n+a}) \tag{10}$$

and

$$\overset{*}{p}{}_{kj}^{n+a} = \sum_{i=j-M}^{j+M-k} \frac{1}{h^k} \Delta_+^k (a\overset{*}{\lambda}{}_{ki}^{1} v_i^{n+1} + \overset{*}{\lambda}{}_{ki}^{11} v_i^n + \overset{*}{\lambda}{}_{ki}^{111} v_i^{n-1}),$$

$$\bar{p}_{kj}^{n+a} = \sum_{i=j-M}^{j+M-k} \frac{1}{h^k} \Delta_+^k (\bar{\lambda}_{ki}^{1} v_i^{n+1} + \bar{\lambda}_{ki}^{11} v_i^{n-1+a}), \qquad k=0,1,\cdots,2M-1, \tag{11}$$

$$\overset{*}{q}{}_{kj}^{n+a} = \sum_{i=j-M}^{j+M-k} \frac{1}{h^k} \Delta_+^k \left[a\overset{*}{u}{}^{\prime}_{ki} \left(\frac{v_i^{n+1} - v_i^n}{\Delta t} \right) + \overset{*}{\mu}{}_{ki}^{11} \left(\frac{v_i^n - v_i^{n-1}}{\Delta t} \right) \right],$$

$$\bar{q}_{kj}^{n+a} = \sum_{i=j-M}^{j+M-k} \mu_{ki} \frac{\Delta_+^k (v_i^{n+a} - v_i^{-k+a})}{h^k \Delta t}, \qquad k=0,1,\cdots,M-1; \tag{12}$$

here all λ's and μ's are constants and $\sum_{i=j-M}^{j+M-k} (a\overset{*}{\lambda}{}'_{ki} + \overset{*}{\lambda}{}''_{ki} + \overset{*}{\lambda}{}'''_{ki}) = \sum_{i=j-M}^{j+M-k} (\bar{\lambda}'_{ki} + \bar{\lambda}''_{ki}) = 1$ $(k=0, 1,\cdots,2M-1)$ and $\sum_{i=j-M}^{j+M-k} (a\overset{*}{\mu}{}'_{ki} + \overset{*}{\mu}{}''_{ki}) = \sum_{i=j-M}^{j+M-k} \mu_{ki} = 1$ $(k=0,1,\cdots,M-1)$ also $j-M \leqslant j* \leqslant j+M$, $n-1 \leqslant n* \leqslant n+1$. The corresponding finite difference boundary conditions are

$$\Delta_+^k v_0^n = \Delta_-^k v_J^n = 0 \qquad (k=0,1,\cdots,M-1;\ J=2,\cdots,N) \tag{2}_h$$

The corresponding finite difference initial conditions

$$v_j^0 = \bar{\varphi}_j,$$

$$\frac{v_j^1 - v_j^0}{\Delta t} = \bar{\psi}_j \qquad (j = 0, 1, \cdots, J) \tag{3}_h$$

where $\bar{\varphi}_j = \varphi(x_j)$, $\bar{\psi}_j = \psi(x_j)$ $(j = M, \cdots, J-M)$ and $\bar{\varphi}_k = \bar{\varphi}_{J-k} = \bar{\psi}_k = \bar{\psi}_{J-k} = 0$ $(k = 0, 1, \cdots, M-1)$

The existence of the solution $v_j^n (j = 0, 1, \cdots, J;\ n = 0, 1, \cdots, N)$ for the finite dfference system $(1)_h$, $(2)_h$ and $(3)_h$ can be proved by the way of the fixed point technique.

LEMMA 4 Under the conditions (I) and (II), for sufficient small Δt, the finite difference system $(1)_h$, $(2)_h$ and $(3)_h$ has at least one solution $v_j^n (j = 0, 1, \cdots, J;\ n = 0, 1, \cdots, N)$, where $0 \leqslant a \leqslant 1$.

§3 Taking the scalar product of the vectors $\frac{\Delta_+^M \Delta_-^M z_j^{n+a}}{h^{2M}} h\Delta t$ and the vector equations $(1)_h$ and then summing up the resulting relations $j = M, \cdots, J-M$, we get

$$(-1)^M \sum_{j=M}^{J-M} \left(\frac{\Delta_+^M \Delta_-^M z_j^{n+a}}{h^{2M}}, z_j^{n+1} - z_j^n \right) h + \Delta t \sum_{j=M}^{J-M} \left(\frac{\Delta_+^M \Delta_-^M z_j^{n+a}}{h^{2M}}, \tilde{A}_j^{n+a} \frac{\Delta_+^M \Delta_-^M z_j^{n+a}}{h^{2M}} \right) h$$

$$= \Delta t \sum_{j=M}^{J-M} \left(\frac{\Delta_+^M \Delta_-^M z_j^{n+a}}{h^{2M}}, F_j^{n+a} \right), \qquad n = 1, 2, \cdots, N-1,$$

where $z_j^{n+1} = \frac{v_j^{n+1} - v_j^n}{\Delta t}$. Then we have

$$\sum_{j=0}^{J-M} \left(\frac{\Delta_+^M z_j^{n+a}}{h^M}, \frac{\Delta_+^M (z_j^{n+1} - z_j^n)}{h^M} \right) h + \frac{a}{2} \Delta t \|\delta_h^{2M} z^{n+a}\|_h^2 \leqslant \frac{\Delta t}{2a} \|F^{n+a}\|_h^2. \tag{13}$$

Here

$$\|F^{n+a}\|_h^2 \leqslant (3M+1) K_1^2 \sum_{j=M}^{J-M} \left\{ \sum_{k=0}^{2M-1} |\bar{p}_{kj}^{n+a}| + \sum_{k=0}^{M-1} |\bar{q}_{kj}^{n+a}| + 1 \right\} h$$

$$\leqslant C_1 \left\{ \sum_{k=0}^{2M-1} (\|\delta_h^k v^{n+a}\|_h^2 + \|\delta_h^k v^{n-1+a}\|_h^2) + \sum_{k=0}^{M-1} \|\delta_h^k z^{n+a}\|_h^2 + 1 \right\}.$$

Using the results of Lemmas 2 and 3 the above expression can be written as

$$\|F^{n+a}\|_h^2 \leqslant MC_1 \varepsilon (\|\delta_h^{2M} v^{n+a}\|_h^2 + \|\delta_h^{2M} v^{n+1+a}\|_h^2)$$
$$+ C_2 (\|\delta_h^M v^{n+a}\|_h^2 + \|\delta_h^M v^{n-1+a}\|_h^2) + C_3 \|\delta_h^M z^{n+a}\|_h^2 + C_4,$$

because $\{v_j^{n+a}\}$ and $\{z_j^{n+a}\}$ satisfy the boundary conditions $(2)_h$ for $n = 0, 1, \cdots, N-1$. Then (13) becomes

$$\|\delta_h^M z^{n+1}\|_h^2 - \|\delta_h^M z^n\|_h^2 + (2a-1) \|\delta_h^M (z^{n+1} - z^n)\|_h^2 + \frac{a\Delta t}{2} \|\delta_h^{2M} z^{n+a}\|_h^2$$

$$\leqslant \Delta t \frac{MC_1}{2a} \varepsilon (\|\delta_h^{2M} v^{n+a}\|_h^2 + \|\delta_h^{2M} v^{n-1+a}\|_h^2) \tag{14}$$

$$+ \Delta t \frac{C_2}{2a} (\|\delta_h^M v^{n+a}\|_h^2 + \|\delta_h^M v^{n-1+a}\|_h^2) + \Delta t \frac{C_3}{2a} \|\delta_h^M z^{n+a}\|_h^2 + \frac{C_4}{2a},$$

where $n=1,2,\cdots,N-1$.

It is evident that

$$\frac{\Delta_+^k v_j^{n+a}}{h^k}=\frac{\Delta_+^k v_j^a}{h^k}+\sum_{m=1}^{n}\frac{\Delta_+^k z_j^{m+a}}{h^k}\Delta t.$$

Summing up for $j=0,1,\cdots,J-K$, we have

$$\|\delta_h^k v^{n+a}\|_h^2\leqslant 2\|\delta_h^k v^a\|_h^2+2T\sum_{m=1}^{n}\|\delta_h^k z^{m+a}\|_h^2\Delta t, \tag{15}$$

where $v_j^a=a\bar{\psi}_j\Delta t+\bar{\varphi}_j$ $(j=0,1,\cdots,J)$ and $k=0,1,\cdots,2M$.

Let us assume $\frac{1}{2}\leqslant a\leqslant 1$.

Substituting (15) for $k=M$ and $2M$ into (14), we get

$$\begin{aligned}&\|\delta_h^M z^{n+1}\|_h^2-\|\delta_h^M z^n\|_h^2+\frac{a\Delta t}{2}\|\delta_h^{2M}z^{n+a}\|_h^2\\&\leqslant 2\Delta t\varepsilon\frac{MC_1T}{a}\left(\sum_{m=1}^{n}\|\delta_h^{2M}z^{n+a}\|_h^2\Delta t+\sum_{m=1}^{n}\|\delta_h^M z^{n+a}\|_h^2\Delta t\right)\\&\quad+\Delta t\frac{C_3}{2a}\|\delta_h^M z^{n+a}\|_h^2\\&\quad+\frac{\Delta t}{2a}(C_4+4MC_1\varepsilon\|\delta_h^{2M}v^a\|_h^2+4C_2\|\delta_h^M v^a\|_h^2).\end{aligned}$$

Summing up to above inequality for n running over the values $1,2,\cdots,n$, then we obtain

$$\begin{aligned}&\|\delta_h^M z^{n+1}\|_h^2-\|\delta_h^M z^1\|_h^2+\frac{a}{2}\sum_{m=1}^{n}\|\delta_h^{2M}z^{m+a}\|_h^2\Delta t\\&\leqslant\varepsilon\left(\frac{2MC_1T^2}{a}\right)\sum_{n=1}^{n}\|\delta_h^{2M}z^{m+a}\|_h^2\Delta t\\&\quad+\left(\frac{C_3T}{2a}+\frac{2MC_1T^2\varepsilon}{a}\right)\sum_{m=1}^{n}\|\delta_h^M z^{m+a}\|_h^2\Delta t+C_5.\end{aligned}$$

Taking ε so small that $\varepsilon\left(\frac{2MC_1T^2}{a}\right)\leqslant\frac{a}{4}$, thus

$$\|\delta_h^M z^{n+1}\|_h^2+\frac{a}{4}\sum_{m=1}^{n}\|\delta_h^{2M}z^{m+a}\|_h^2\Delta t\leqslant C_6\sum_{m=1}^{n+1}\|\delta_h^M z^m\|_h^2+C_7. \tag{16}$$

Denote $W_{n+1}=\sum_{m=1}^{n+1}\|\delta_h^M z^m\|_h^2\Delta t$. So

$$\frac{W_{n+1}-W_n}{\Delta t}\leqslant C_6W_{n+1}+C_7$$

This follows that $\{W_n\}$ is bounded, i. e., $\sum_{m=1}^{n}\|\delta_h^M z^m\|_h^2\Delta t$ is bounded for $n\Delta t\leqslant T$. Then from (16) we have the boundedness of the sequences $\{\|\delta_h^M z^n\|_h^2\}$ and $\{\sum_{m=1}^{n}\|\delta_h^{2M}z^{m+a}\|_h^2\Delta t\}$ for $n\Delta t\leqslant T$.

LEMMA 5 Under the conditions (I), (II) and (III), for sufficient small Δt, the solution $v_j^n (j=0,1,\cdots,J;\ n=0,1,\cdots,N)$ for the finite difference system $(1)_h$, $(2)_h$ and $(3)_h$ has the estimation relations, for any $n\Delta t \leqslant T$,

$$\|\delta_h^k z^n\|_h \leqslant K_5, \qquad k=0,1,\cdots,M \tag{17}$$

and

$$\sum_{m=1}^{n} \|\delta_h^k z^{m+a}\|_h^2 \Delta t \leqslant K_6 \qquad k=0,1,\cdots,2M \tag{18}$$

where $\frac{1}{2} \leqslant a \leqslant 1$, $z_j^{n+1} = \frac{v_j^{n+1} - v_j^n}{\Delta t}$ $(j=0,1,\cdots,J;\ n=1,2,\cdots,N-1)$ and K_5 and K_6 are independent of h and Δt.

COROLLARY Under the conditions of Lemma 5, there are

$$\|\delta_h^k v^n\|_h \leqslant K_7, \qquad k=0,1,\cdots,M \tag{19}$$

$$\|\delta_h^k v^{n+a}\|_h \leqslant K_8, \qquad k=0,1,\cdots,2M, \tag{20}$$

where $\frac{1}{2} \leqslant a \leqslant 1$, $n\Delta t \leqslant T$ and K_7 and K_8 are independent of h and Δt.

From the finite difference system $(1)_h$, we obtain directly the following lemma.

LEMMA 6 Under the conditions of Lemma 5, there is

$$\sum_{n=1}^{m} \left\| \frac{v^{n+1} - 2v^n + v^{n-1}}{\Delta t^2} \right\|_h^2 \Delta t \leqslant K_9, \tag{21}$$

wkere $(m+1)\Delta t \leqslant T$ and K_9 is independent of h and Δt.

Using the Lemma 1 over and over again on the above estimations; we have the following lemma.

LEMMA 7 Uuder the conditions of Lemma 5, there are the estimation relations:

$$\max_{j=0,1,\cdots,J-k} |\Delta_+^k v_j^{n+1}| \leqslant K_{10} h^k \qquad (k=0,1,\cdots,M-1); \tag{22}$$

$$\max_{j=0,1,\cdots,J-M} |\Delta_+^M v_j^{n+1}| \leqslant K_{11} h^{M-\frac{1}{2}}; \tag{23}$$

$$\max_{j=0,1,\cdots,J-k} |\Delta_+^k v_j^{n+a}| \leqslant K_{12} h^k \qquad (k=0,1,\cdots,2M-1); \tag{24}$$

$$\max_{j=0,1,\cdots,J-2M} |\Delta_+^{2M} v_j^{n+a}| \leqslant K_{13} h^{2M-\frac{1}{2}}, \tag{25}$$

where K's are independent of $(n=0,1,\cdots,N-1)$, h and Δt;

$$\max_{n=0,1,\cdots,N-1} |\Delta_+^k (v_j^{n+1} - v_j^n)| \leqslant K_{14} h^k \Delta t \qquad (k=0,1,\cdots,M-1); \tag{26}$$

$$\max_{n=0,1,\cdots,N-1} |\Delta_+^M (v_j^{n+1} - v_j^n)| \leqslant K_{15} h^{M-\frac{1}{2}} \Delta t; \tag{27}$$

$$\max_{n=0,1,\cdots,N-1} |\Delta_+^k (v_j^{n+a} - v_j^{n-1+a})| K_{16} h^k \Delta t^{\frac{1}{2}}, \quad (k=0,1,\cdots,2M-1); \tag{28}$$

$$\max_{n=0,1,\cdots,N-1} |\Delta_+^k (v_j^{n+1} - 2v_j^n + v_j^{n-1})| \leqslant K_{17} h^k \Delta t^{\frac{3}{2} - \frac{k+\frac{1}{2}}{2M}}, \quad (k=0,1,\cdots,M-1), \tag{29}$$

where K's are independent of $j(=0,1,\cdots,J-k)$, h and Δt.

Finally, using the estimation relations in Lemmas 5, 6 and 7, we can prove the following convergence theorem of the finite difference scheme $(1)_h$, $(2)_h$ and $(3)_h$.

THEOREM Suppose that the conditions (I), (II), (III) and (IV) are satisfied. Then the m-dimensional vector valued discrete function v_j^n $(j=0,1,\cdots,J;\ n=0,1,\cdots,N)$ of the finite difference system $(1)_h$, $(2)_h$ anh $(3)_h$ converges to a limit $u(x,t)\in Z$ as $h^2+\Delta t^2\to 0$. And $u(x,t)$ satisfies the nonlinear pseudo-hyperbolic system (1) in generalized sense and satisfies the boundary conditions (2) and the initial condition (3) in classical sense.

References

[1] Pao, C. V., A Mixed Initial Boundary Value Problem Arising in Neurohypsiology, *J. Math. Anal.* Appl., 52 (1975), 105—119.

[2] Гоьлдъерг, З. А. О Параметриическом усилении стоящих волн В жидкости, *ДАН СССР*, 201 (1971), 304—306.

[3] Davis, P. L., A Quasilinear Hyperbolic and Related Third Order Equations, *J. Math. Anal. Appl.*, 51 (1975), 596—606.

[4] Caughey, T. K. & J. Ellison, Existence, Uniqueness and Stability of Solutions of a Class of Nonlinear Partial Differential Equations, *J. Math. Anal. Appl.*, 51 (1975), 1—32.

[5] Andrews, G., On Existence of the Equation $u_{tt}=u_{xxt}+(\sigma(u_x))_x$, *J. Diff. Equations*, 35 (1980), 200—231.

[6] Yukiyoshi Ebihare, On Some Nonlinear Evolution Equations with the Strong Dissipation, *J. Diff. Equations*, 30 (1979), 149—164.

[7] Zhou Yu-lin & Fu Hong-yuan, The Nonlinear Pseudo-Hyperbolic System of Higher Order, *J. Math. Research & Exposition*, 2 (1982), 37—50.

[8] Zhou Yu-lin, Boundary Value Problems for Some Nonliear Evolutional Systems of Partial Differential Equations, (to appear).

[9] Zhou Yu-lin, Finite Difference Method of First Boundary Problem for the Quasilinear Parabolic Systems, (to appear).