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Packing Spheres and Uniformly Nonsquare Property in Orlicz Sequence Space*

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Recently, the reserches on geometric characteristic in Orlicz space have made great progresses. In the aspects of strict convex and uniformly convex, H. W. Milnes[1]. M. M. Rao[2]. Wu Congxin etc[3], V. Akimovic[4] and others have obtained a series of satisfying results; In other geometric characteristics, C. E. Cleaver[5]. K. Sundaresan and one the writers of this paper[6] have also obtained some considerably significant results. In this paper, we shall first discuss packing spheres and uniformly non-square in Orlicz function spaces and Orlicz sequence spaces, then, we shall use these result to prove that packing spheres critical value Λ can be regarded as a characteristic number of space being uniformly non-square.

Definition 1 Packing spheres critical value Λ of Banach space X is a number such that if $r\leqslant\Lambda$, then a infinite number of spheres of radius r can be packed in the unit ball U, and if $r>\Lambda$, only a finit number of spheres of radius r can be packed in $B(X)$. The problem of packing spheres is just to find out the magnitube of the value Λ. It attracts great attention because of its clear geometric characteristics. Rankin[10][11] Burlack, Robertson. Wells. Willians[12] and Kottman[14] published, early or late, results of this kind in their thesis or monographs of themselves. In 1976, C. E. Cleaver studied separable Orlicz spaces and found upper and lower bounds of Λ. His results almast covers all results which have been so far obtained, one of the writers of this paper has obtained a exact value Λ of separable Orlicz sequence, theorem 1 in this paper solves the packing spheres problem in non-separable Orlicz sequence spaces.

Theorem 1 The packing spheres critical value Λ in non-separable Orlicz function and sequence spaces are both equal to $\frac{1}{2}$.

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In thesis[14], Kottman has pointed out that the PSV (packing spheres critical value) of l^∞ $\Lambda=\frac{1}{2}$. At the same time, it is easily proved that the PSV, Λ of L is $\frac{1}{2}$. So one would guess if PSV of each non-separable Banach space equals $\frac{1}{2}$, In this paper, we give a inverse example and thus prove if Hilbert space H_c is not separable, then $\Lambda=1/(1+\sqrt{2})$.

Definition 2 A normed linear space is uniformly non-square if and only if there is a pasitive number δ such that for any two elements x and y in the unit ball, $\|\frac{1}{2}(x+y)\|>1-\delta$ and $\|\frac{1}{2}(x-y)\|>1-\delta$ can't both be true.

Definition 2 was given by James, the writer of thesis[13]. In 1966. K. Sunderasan studied uniformly non-square Orlicz function spaces, and obtained some sufficient and necessary conditions for Orlicz function spaces to become uniformly non-square, but these are all on the condition: $M(u)$ satisfies Δ_2 condition for large value of u. Theorem 2 in this paper showed that these conditions can be removed.

Theorem 2: If the Orlicz function space L_M^* produced by $M(u)$ is uniformly non-square, then $M(u)$ satisfies Δ_2 condition for large value of u; If the Orlicz sequence space L_M produced by $M(u)$ is uniformly non-square, the $M(u)$ satisfies Δ_2 condition for small value of u.

Theorem 2 can be writen as follows:

Theorem 2' Uniformly non-square Orlicz function space and Orlicz sequence space are both separable.

In order to prove the key theorem of this paper, we need to prove the following lemma:

Lemma: If $M(u)$ satisfies Δ_2 condition for small value of u, but l_m is not uniformly non-square, and $x^{(n)}$、$y^{(n)}\in S(l_m)$, $x^{(n)}=(x_i^{(n)})_{i=1}^\infty$, $y^{(n)}=y_i^{(n)})_{i=1}^\infty$, such that

$\|\frac{1}{2}(x^{(n)}\pm y^{(n)})\|\geqslant 1-\frac{1}{n}\qquad (n=1,2,\cdots\cdots)$.

Let $Z_1^{(n)}=\{i: |x_i^{(n)}|\geqslant|y_i^{(n)}|\}$, $\qquad Z_2^{(n)}=\{i: |x_i^{(n)}|<|y_i^{(n)}|\}$

then

$$\lim_{n\to\infty}[\sum_{i\in Z_1^{(n)}}M(y_i^{(n)})+\sum_{i\in Z_2^{(n)}}M(x_i^{(n)})]=0.$$

With theorem 1, theorem 2, and lemma, the main propasition is obtained:

Theorem 3 Let l_m be Orlicz sequence space produced by $M(u)$, then: l_m is uniformly non square if and only if $\Lambda_M<\frac{1}{2}$.

Kottman have pointed out that (see thesis [14] Th3, 2) $P(n, X)<\frac{1}{2}$ contain reflexive property for Banach spaces, and given a inverse example to explain the reason why it is not true that $P(\aleph_0, X)<\frac{1}{2}$, i,e $\Lambda_X<\frac{1}{2}$ contain reflexive property for a Banach space.

But using theorem 3 we can obtain:

Corollary 1: $\Lambda_M<\frac{1}{2}$, then l_m is reflexive.

Corollary 2: If l_m is uniformly converx, then $\Lambda_M<\frac{1}{2}$.

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