Lemma 6. If $p\geqslant 7$ is odd, B can win in the game.

Proof: Supose A colours edge xy, B can colour xy', $Y=V-\{x\}$, where $V=V(K_p)$. If A colours xy'', then B colours $xy'''\cdots$, and so on. Because $|Y|=p-1$ is even, B can force A first colours in Y, according to Lemma 4 and Lemma 5, B can ensure that G (Y) has an H-path.

Obviously, we have $d_4(x)=\frac{p-1}{2}$. Thus we can easily see that G has an H-path. By all the Lemmas above, we can get the following theorem.

Theorem If $p\geqslant 5$, B can win in the game. If $p=2$, 3, B can win trivally, but for $p=4$, B will lose.

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Block diagonal dominance of matrix and spectral inclusion regions*

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Suppose that comlex matrix A or order n is partitioned as

$$A=\begin{bmatrix} A_{11}\cdots A_{NN} \\ \cdots\cdots\cdots\cdots \vdots \\ A_{N1}\cdots A_{NN} \end{bmatrix} \qquad (1)$$

where the diagonal submatrices A_{ii} are square of order n_i $(1\leqslant i\leqslant N)$. If each A_{ii} is nonsingular and satisfies

$$\sum_{\substack{j=1\\ j\neq i}}^{N} \| A_{ii}^{-1}A_{ij} \| \leqslant 1, \qquad 1\leqslant i\leqslant N. \qquad (2)$$

thon A is called quasi-block diagonally dominant. Specially, if strictly inequality in (2) is valid for all $1\leqslant i\leqslant N$ then A is called quasi-block strictly diagonally dominant. If strict inequality in (2) is valid for at least one i $(1\leqslant i\leqslant N)$ and

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A is block irreducible (i. e. $B=(\|A_{ij}\|)_{N\times N}$ isrrieducible). then A is called quasi-block irreducibe diagonally dominant. If B is weakly irreduable (i. e. every vertex of Γ (B) belongs to some circuit of Γ (B), where Γ (B) denotes directed graph of B), then A is called weakly irreducible.

Theorem 1. Let $n\times u$ complex matrix A be partitioned as in (1) which is either quasi- block strictly diagonally dominant or quasi-block irreducible diagonally dominant, then A is nonsingular.

Corollary 1. Let $n\times n$ complex matrix A be partitioned as in (1). Then the eigenvalues of A lie in the union of the regions

$$G_i=\{z, \sum_{\substack{j=1\\ j\neq i}} \|(zI-A_{ii})^{-1}\|\geqslant 1\}, \qquad 1\leqslant i\leqslant N.$$

Corollary 2. Let $n\times n$ complex A be partitioned as in (1) which is block irreducible, λ is any eigenvalue of A. If λ is a boundary point of $G=\bigcup_{i=1}^{N} G_i$, then it is a boundary point of each set G_i, $1\leqslant i\leqslant N$.

Theorem 2. Let $n\times n$ complex A be partitioned as in (1) which is block weakly irreducible ($1<N$), S(B) denotes the set of all circuits of directed graph of B. If the

$$\prod_{i\in v}\left(\sum_{\substack{j=1\\ j\neq i}}^{N} \|A_{ii}^{-1}A_{ij}\|\right)<1 \qquad (v\in S(B)),$$

then A is nonsingular.

Corollary 3. Let $n\times n$ complex matrix A be partitioned as in (1) which is block weakly irreducible ($1<N$). Then the eigenvalues of A lie in the union of the regions

$$D_v=\left\{z:\prod_{i\in v}\left(\sum_{\substack{j=1\\ j\neq i}}^{N} \|(zI-A_{ii})^{-1}A_{ij}\|\right)\geqslant 1\right\} \quad (v\in S(B)).$$

Theorem 3. Let $n\times n$ complex matrix A be partitioned as in (1), $|\lambda|_{min}$ denotes the small eigenvalue of A which is according to modules. Then

$$|\lambda|_{min}\geqslant \min_{1\leqslant i\leqslant N}\left\{(\|A_{ii}^{-1}\|)^{-1}\left(1-\sum_{\substack{j=1\\ j\neq i}}^{N}\|A_{ii}^{-1}A_{ij}\|\right)\right\}$$

The Theorem 1 and Theoemr 2 in this parper are generalizaions of the Theorem 1 in parper of Fiengold and Varga (Pac. J. Math., 12(1962), 1241-1250) and the Theorem 2.3 in parer of Brualdi (Linear Multilin Alg, 11 (1982), 147-165), respectively.

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