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# Remarks on minimal hypersur faces of sphere with constant scalar curvature*

*Wu Chuanxi* (吴传喜)

(Hubei University)

Let M be a n-dimension closed minimally immersed hypersurface in the unit sphere $S^{n+1}$, and let $h$ denote the second fundamental form of M. We denote the square of the length of h by $S$. Then we have $S=n(n-1)-R$, where $R$ is the scalar curvature of M, which shows that $S$ is intrinsic. In particular, $S$ is constant if and only if M has constant scalar curvature. It is well-known that if $0\leqslant S\leqslant n$, then $S\equiv 0$ or $S\equiv n$ (J.Simons[1]). The minimal hypersurfaces with $S=0$ are the equatorial n-spheres in $S^{n+1}$, S.S. Chern, Do Chrmo and S. Kobayashi[2] and B. Lawson[3] proved independently that minimal hypersurfaces with $S=n$ are the only Clifford tori. Naturally, S.S. Chern asked the following question[2]: Is there a next large value for $S$, and if so what is it ? C. K. Peng and C. L. Terng[4] obtained the following important theorem, giving a partial answer to this problem.

**Theorem**[4]. Let M be a closed minimally immersed hypersurface in $S^{n+1}$ with $S=$constant. If $S>n$, Then $S>n+\frac{1}{12n}$.

In this paper, by some elementary inequality we improved the theorem as follows.

**Theorem 1**. Let M be a closed minimally immersed hypersurface in $S^{n+1}$ with $S=$ constant. If $S>n$, then $S>n+\frac{1}{3(\sqrt{17}-1)n}$.

Moreover, we proved

**Theorem 2**. Let M be a closed minimally immersed hypersurface in $S^{n+1}$ with constant principle curvature. If $f_4<\frac{49}{18}+9n-4n^2+\frac{f_3^2}{5n}$, where $f_m$ denotes the $m$th symmetric function of the principle curvature, then $S=0, n$.

## References

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