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A Characterization of a p-Uniformly Convex Space

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V.I.Istrățescu[1,p.102] introduces the following notions: A Banach space E is said to be p-uniformly convex ($p\geqslant 2$) if the modulus of convexity of E satisfies the inequality

$$\delta_E(t)\geqslant Ct^p,\qquad 0\leqslant t\leqslant 2,$$

for some positive constant C. A Banach space E is said to be q-uniformly smooth ($1<q\leqslant 2$) if the modulus of smoothness of E satisfies the inequality

$$\rho_E(s)\leqslant ds^q\qquad \forall\, s\geqslant 0.$$

for some positive constant d, where

$$\delta_E(t)=\inf\{1-\left\|\frac{x+y}{2}\right\|:\|x\|=\|y\|=1,\ \|x-y\|=t\},$$

$$\rho_E(s)=\frac{1}{2}\sup\{\|x+y\|+\|x-y\|-2:\|x\|=1,\ \|y\|=s\}.$$

In this note we prove the following conclusion:

Theorem A Banach space E is p-uniformly convex if and only if E^* is q-uniformly smooth; E is q-uniformly smooth if and only if E^* is p-uniformly convex, where $p\geqslant 2$, $1<q\leqslant 2$, $\frac{1}{p}+\frac{1}{q}=1$.

References

[1] Istrățescu, V.I., Strict convexity and complex strict convexity, Lecture Notes in Pure and Appl.Math., vol.89, Marcel Dekker Inc., New York, 1984.

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