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A Class of singular pseudo-homogeneous pseudodifferential operator

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In this paper, we considered a class of weighted Sobolev space and in an uniform way researched Fourier-Bessel singular pseudodifferential operator (below denoted by SP_SDO) associated with pseudo-homogeneous (include homogeneous) symbol. Let

$$H_\nu^{s,q}=\{u(x,t)\in S_{e\nu}^{q}(R_{n+1}^{+}),\ \|u\|_{s.q.\nu}^{2}=\int_{R_{n+1}^{+}}(1+|\xi|^{2}+\tau^{2q})^{s}|\hat{u}(\xi,\tau)|^{2}\tau^{2\nu+1}d\xi d\tau<+\infty\}$$

where $u(x,t)$ denots even function of t and satisfy:

$$|\partial_x^\alpha B_t^\beta u(x,t)|\leqslant C(|+|x|+|t|^q)^{-\gamma},\ x\in R^n,\ t\in R^+,\ \alpha\in Z_+ \text{ and } \gamma>0 \text{ arbitrary},\ 0<q\leqslant 1 \text{ is fixed}.$$

B_t denots Bessel operator, $\hat{u}(\xi,\tau)$ is the Fourier Bessel trasform of $u(x,t)$. Easy to prove, $H_\nu^{s,q}$ is a Hilbert space, by same way $H_{o,\nu}^{s,q}$, $H_{loc,\nu}^{s,q}$ Can be defined.

Introduce a class of SP_SDO A: $Au(x,t)=C\int e^{ix.\xi}j_\nu(t\tau)a(x,t,\xi,\tau)\tau\hat{u}(\xi,\tau)\tau^{2\nu+1}d\xi d\tau$ [2], where $a\in\tilde{S}_{e\nu}^{m,q}=\{a\in C^\infty$ is even function of t,τ.

$$|\partial_x^\alpha\partial_\xi^\beta B_t^\gamma B_\tau^\delta a|\leqslant C(1+|\xi|+|\tau|^q)^{m-|\beta|-\gamma}(1+|\tau|^q)^{1-2\delta}\ \forall\alpha,\beta\in Z_+^n,\gamma,\delta\in Z_+\}.$$

We get following result:

1) $A: C_{e\nu,o}^\infty(\Omega)\to C_{e\nu}^\infty$ is boundary continuous mapping and can be extended to the continaous mapping from the $H_{o.\nu}^{s,q}(\Omega)$ to $H_{loc.\nu}^{s-m,q}(\Omega)$, the definition of $C_{e\nu.o}^\infty$, $C_{e\nu}^\infty$ can be fined in [2].

2) Give $a_j\in\tilde{S}_{e\nu}^{m_j,q}$, $m_j\downarrow-\infty(j\to+\infty)$ then exist function $a(x,t.\xi,\tau)\in\tilde{S}_{e\nu}^{m_o.q}$ such that $a\sim\sum_1^\infty a_j$ and $a(x,t,\xi,\tau)$ is unique in mod $\tilde{S}_{e\nu}^{-\infty}$.

3) if $A=\sum_{|\alpha|+2bj=2m}a_{\alpha j}(x,B_t,D_x)B_t^jD_x^\alpha$, $a_{\alpha j}$ is a standard pseudodifferential operator for x $a_{\alpha j}(x,\tau,\xi)$ is homogeneous of order zero of (ξ,τ), and satisfy:

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$\Sigma a_{\alpha j}(x, r\tau, r\xi)(r^b\tau)^{2j}(r\xi)^\alpha = r^{2m}\Sigma a_{\alpha j}(x, \tau, \xi)\tau^{2j}\xi^\alpha \neq 0$ $r > 0$. $\forall (x . t . \xi . \tau) \in \Omega . \times R_{n+1} \backslash 0$, $b \geqslant 1$, then exist the parametrix of A the class of $\widetilde{S}_{ev}^{-2m, \frac{1}{b}}$. Especially when b=1.It is rightly B-elliptic, parametrix when b=1. $\nu = -\frac{1}{2}$, it is rightly standard elliptic parametrix .

References

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