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Several Convexity Statements in Stochastic Programming (I)*

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There is a Kind of so-called chance constrained programming problems in stochastic programming. One of the main subjects of this kind of problems is whether the constrained set $X(a)\triangleq\{x|P(\omega|A(\omega)x\geq b(\omega))\geq a,\ x\in X\}$ or $X_i(a_i)\triangleq\{x|P(\omega|A_i(\omega)x\geq b_i(\omega))a_i,\ x\in X\}$ is convex.

In this letter, we discuss the convexity statement if all the elements of the fist column of $A(\omega)$ are independent random variables with known Weibull distributions and others are fixed. Suppose density function for Weibull distribution

$$f(x)=\begin{cases}\mu\nu x^{\mu-1}e^{-\nu x^{\mu}}, & x>0,\\ 0, & x\leq 0,\end{cases}\quad (\text{here } \mu>0 \text{ and } \nu>0 \text{ are constants}),$$

it should be observed that Weibull distribution becomes exponential distribution if $\mu=1$. We have obtained the following:

Theorem 1 If all the elements of the first column of $A(\omega)$ are independent random variables with known exponential distributions and others are fixed, $X\subset\{x_1\geq 0\}$ or $X\subset\{x_1\leq 0\}$ then $X(a)$ is convex.

Theorem 2 If all the elements of the first column of $A(\omega)$ are independent random varibles with known Weibull distributions ($\mu\neq 1$) and others are fixed, then

(i) Suppose $X\subset\{x_1\geq 0\}$ and $\mu>1$, then $X(a)$ is convex.

(ii) Suppose $X\subset\{x\leq 0\}$ and $0<\mu<1$, then $X(a)$ is convex.

References

[1] Peter Kall, Stochastic Linear Programming, Springer-Verlag, Berlin, Herdolberg, New-York, 1976.

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