J.Mith. Res. & Exposition, 1987, No.2

Approximation-Transforming Theory and Pansystems Approximation Theory(I)*

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Approximation transforming theory (ATT) is a transtopical research on approximation and related mathematical disciplines from the viewpoint of pansystems, and its extension to the generalized approximation to generalized systems and the so-called pansystems concepts is called pansystems approximation theory (PAT). The series of articles presented is a complement and development of the author's former work.

Theorem 1. If $D(\subset R^m)$ is measurable, $0<p\leq q<s<\infty$, $g\in L^s(D)$, then $\|g\|_q\leq\|g\|_p^a\|g\|_s^b$, where $\|*\|_q$ means the norm of $L^q(D)$, etc, and $a=s(q-p)/[q(s-p)]$, $b=p(s-q)/[q(s-p)]$.

Theorem 2. If D has conic property and is of open set, $g\in W_s^{(l)}(D)$, $ls<m$, $s>1$, then $\|g\|_q\leq c\|g\|_{(l)s}^a\|g\|_p^b$, where $\|*\|_{(l)s}$ is the norm in $W_s^{(l)}(D)$, $q\geq p\geq 1$, c is independent of g, and $a=ms(q-p)/[q(lsp-mp+mq)]$, $b=[(lsp-mp)(q-p)/[q(lsp-mp+mq)]]+p/q$.

Theorem 3. If $D(\subset R^2)$ is open domain, its boundary B is rectifiable, $H_\lambda=\{g\mid g\in C(\overline{D}),\ g|_B=0,\ \partial g\in C(D),\ K\}$,

$$K:\ \|g\|=\left\{\iint_D[A_1(\frac{\partial g}{\partial x})^2+A_2(\frac{\partial g}{\partial y})^2+A_3g^2]\mathrm{d}x\mathrm{d}y\right\}^{1/2}<\infty,\left\{\int_l(\frac{\partial g}{\partial y})^2\mathrm{d}y\right\}^{1/2}\leq\lambda,$$

where $A_i>0$ and belong to $C(\overline{D})$, l is any segment parallel to y-axis in D, then $g\in H_\lambda$ implies $\|g\|_q\leq c\|g\|[\log(\lambda/\|g\|)]^a$, $q\geq 2$, $a=(q-2)/q$, and c is independent of g and λ.

By using this sort of inequalities and the PMS transforming principles we can derive a series of error transforming theorems and indirect theorems of approximation (see[1]).

References

[1] Wu Xuemou, Approximation Transforming Theory and Pansystems Concepts in Mathematics, Hunan Press of Science and Technology, China, 1984.

[2] Wu Xuemou, Science Exploration, 1, 2, 4 (1982), 1, 4 (1983), 1 (1985).

[3] Wu Xuemou, Science Exploration, 3 (1986).

* Received Jan. 15, 1987.