J.Math.Res.& Exposition, 1987, No.3

A Counter Example on Faudree-Schelp Conjecture

Liu Bo-Lian (柳柏濂)
(South China Normal University)

Let simple graph $G=(V, E)$, $V=n$, $E=m$. If there exists a path containing i vertices connecting u and v in V, then property $P_i(u,v)$ will be said to told. For $2\le i\le n$, let S_i be the set of all unordered pairs of distinct u and v for which property $P_i(u,v)$ holds, and Let S_1 be the set of all unordered pairs of vertices which are not connected by any path. A graph G satisfies property P_i if $|S_i|=n(n-1)/2$.

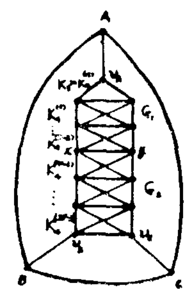
In the International Conference on the Theory and Application of Graph (1976), R.J.Faudree and R.H. Schel[1] offered following conjecture: Property P_n implies property P_i for each i, $n/2+1\le i\le n$, in a graph of order n.

Now, we give following a kind of graph, which shows that Faudree-Schelp cnnjecture is not true.

Let's construct graph G' of order n, $n\equiv 0$ mod 4 ($n\ge 8$)(see Fig.) $|V(G')|=2(2r-3)+6=4r$, $(r\ge 2)$.
We easily examine that P_n holds in G', $n=4r$.

Let $K_3=K_4^{(0)}$. Denote two vertices in common of $K_4^{(r-2)}$ and $K_4^{(r-1)}(r\ge 2)$ by x, y.

Let G_1 be the subgraph censisting of $K_4^{(0)}, K_4^{(1)}, \cdots, K_4^{(r-2)}$, G_2 be the subgraph consisting of $K_4^{(r-1)} K_4^{(r)}, \cdots, K_4^{(2r-3)}(r\ge 2)$.


G' $(r\ge 2)$

Since $|V(G_1)|=2(r-2)+3=2r-1=n/2-1$,
$|V(G_2)|=2(r-1)+2=2r=n/2$

therefore, any path $P_{n/2+1}(x,y)$ must pass two points of $\{A, B, C\}$ at least, i.e. $P_{n/2+1}(x,y)$ must pass two points of $\{V_A, V_B, V_C\}$ at least.

According to the contruction of G' the shortest path from x(or y) to each of $\{V_A, V_B, V_C\}$ contains $r=n/4$ vertices.

Note that any pair of $\{V_A, V_B, V_C\}$ has no adjacent vertice in common in $G'-(G_1\cup G_2)$. The path passing two points of $\{A, B, C\}$ at least have $n/4+n/4+2=n/2+2$ vertices at least. Thus there is no $P_{n/2+1}(x,y)$ in G'.

Obviously, for $n\equiv 0$ mod $4(n\ge 8)$ we can construct the graph of order n, which Faudree-Schelp conjecture doesn't hold.

References

[1] R.J.Faudree and R.H.Schelp, Various Length Paths in Graphs, in Theory and Applications of Graphs, Proceedings, Michign, May 11—15 1976, pp. 160—173.