J.Math.Res.& Exposition, 1988, No. 1

Approximation-Transforming Theory and Pansystems Approximation Theory (IV)*

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Theorem 1 If $1 \leqslant p \leqslant \infty$, $f \in W_p^{(l)}(D)$, then $\omega_k(\delta, f, W_p^{(l)}(D)) \leqslant c(\|f\|_{(l)_p})$, if $f \in C^{(k+l)}(\overline{D})$, then $\omega_k(\delta, f, W_p^{(l)}(D)) \leqslant c(\delta^k \max_{D^{(k)}} \|D^{(k)} f\|_{(\ )_p})$, where c is independent of $\delta \geqslant 0$ and f.

Theorem 2 If $f \in W_p^{(r)} H_M^{(a)}([a, b])$ is of period $b-a < \infty$, then $\|f\|_{(s)t} \leqslant cM^d \|f\|^e_{(u)v}$, where $d = \delta/\theta$, $e = (\theta - \delta)/\theta$, $p > 1$, $t \geqslant v \geqslant 1$, $r > s \geqslant u$, $\delta = s - u +$ $+ (\frac{1}{v} - \frac{1}{s})$, $\theta = r + a - u - \max(0, \frac{1}{p} - \frac{1}{v})$, and c is independent of M and f.

Theorem 3 If $f \in C[-1, 1]$, the optimal approximation degree for f by polynomial of n-order is $a(n)$, and $S_n(f)$ is the n-partial sum of Fourier series of f according to the Legendre polgnomial expansion, then $\|S_n(f)\|_C \leqslant (n+1)^2 \cdot \|f\|_C$, $\|f - S_n(f)\|_C = O(n^{\frac{1}{2}} a(n))$.

Similar results can be obtained for Fejer and Chebershev interpolation polynomials.

Theorem 4 If $f(x) \in C[-1, 1]$, p_n is polynomial of n-order, $\|f - p_n\|_C \leqslant a(n) \downarrow 0$, $\int^{\infty} a(t) dt < \infty$, then $f'(x) \in C(-1, 1)$, and for $u > 1$, $|f'(x) - p_n'(x)| \leqslant \frac{4}{(\sqrt{1 - x^2}) \log u} \int_n^{\infty} a(\frac{t}{u}) dt$, $|x| < 1$.

Theorem 5 If T_n is trigonomet polynomial of n-order, then $\|T_n^{(k)}\|_s \leqslant cn^a \cdot \|T_n\|_r$, where $a = \frac{1}{r} - \frac{1}{s} + k$, $c = 1$ for $1 \leqslant s = r \leqslant \infty$; $c = 2$ for $o < r \leqslant s \leqslant \infty$, $k = 2$, or for $1 \leqslant s \leqslant \infty$, $0 < r < s$.

*Received Apr. 23, 198 .