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Propositional Calculus System of Medium Logic (Ⅱ)

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This paper is a continuation of [1]、[2]、[4]. In this paper, we shall continue to constitute the formal theorems and the important substituting theorem (see theorem 16 in this paper) of the propositional calculus system of medium logic MP. The order numbers of following formal theorems follow those in reference [4]

Theorem 11 MP:

[1] $A\to B,\ A\to\neg B \vdash \neg A$

[2] If $*B$, ΔB are different ones of B, $\sim B$, $\rceil B$, then $A\to *B$, $A\to \Delta B \vdash \neg A$

[3] $A\to B, \rceil A\to B \vdash B$

[4] $A\to B, \rceil A\to B \vdash B$

[5] If $\Gamma, A\vdash B$ and $\Gamma, \rceil A \vdash B$ and $\Gamma, \sim A \vdash B$. Then $\Gamma \vdash B$.

Proof of [1]: Using($\to$) and ($\neg_+$)

Proof of [2]: Let $*B$ be $\sim B$ and ΔB be B, we are to prove

$$A\to\sim B,\ A\to\rceil B \vdash \neg A$$

(1) $A\to\sim B$

(2) □□□$A\to\rceil B$

(3) □□□□□A

(4) □□□□□$\sim B$ (1)(3)($\to_-$)

(5) □□□□□$\neg\rceil B$ (4)(Y)

(6) □□□□□$\rceil B$ (2)(3)($\to_-$)

(7) □□□ $\neg A$ (5)(6)($\neg_+$)

Proof of [3]:

(1) $A\to B$

(2) □□ $\neg A\to B$

(3) □□□□ $\neg B$

(4) □□□□ $\neg A$ (1)(3)(MP7[4])

(5) □□□□B (2)(4)($\to_-$)

(6) □□B (3)(5)($\neg$)

Proof of [4]: Similar to [3] and using theorem 7[1].

Proof of [5]:

(1) Γ

(2) □□□□□□□A

(3) □□□□□□□B (1)(2)hypotheses

(4) □□□□□□$\sim A$

(5) □□□□□□B (1)(4)hypotheses

(6) $A\to B$ (3)(5)($\to_+$)

(7) □□□□ $\rceil A$

(8) □□□□B (1)(7)hypotheses

(9) □□$\sim\rceil A$

(10) □□$\sim A$ (9)(MP4[1])

(11) □□B (1)(10)hypotheses

(12) $\rceil A\to B$ (8)(11)($\to_+$)

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(13) B (6)(12)(MP11[4])

Theorm 12 MP:

[1] $\sim A, \sim B \vdash \sim(A \to B)$

[2] $\sim A, \bar{\neg} B \vdash \sim(A \to B)$

[3] $A, \sim B \vdash \sim(A \to B)$

[4] $\sim(A \to B), A \vdash \sim B$

[5] $\sim(A \to B), \bar{\neg} B \vdash \sim A$

Proof of [1]:

(1) $\sim A$

(2) □$\sim B$

(3) □□□□□$A \to B$

(4) □□□□□B (1)(3)($\to_{-}$)

(5) □□□□□ $\neg B$ (2)($Y_{\sim}$)

(6) □□$\bar{\neg}(A \to B)$

(7) □□A (6)($\bar{\neg}_{\to}$)

(8) □□$\neg A$ (1)($Y_{\sim}$)

(9) □$\sim(A \to B)$ (4)(5)(7)(8)(TH6[2])

Proof of [2]、[3]: Similar to [1] and using ($Y_{\bar{\neg}}$).

Proof of [4]:

(1) $\sim(A \to B)$

(2) □□□A

(3) □□□□□□□B

(4) □□□□□□□$A \to B$ (3)(MP1[2])

(5) □□□□□□□ $\neg(A \to B)$ (1)($Y_{\sim}$)

(6) □□□□$\neg B$

(7) □□□□ $\bar{\neg}(A \to B)$ (2)(6)($\bar{\neg}_{\to}$)

(8) □□□□$\neg\sim(A \to B)$ (7)($Y_{\bar{\neg}}$)

(9) □□□$\sim B$ (4)(5)(1)(8)(MP6[2])

Proof of [5]:

(1) $\sim(A \to B)$

(2) □□□$\bar{\neg} B$

(3) □□□□□□□□□A

(4) □□□□□□□□□$\bar{\neg}(A \to B)$ (2)(3)($\bar{\neg}_{\to}$)

(5) □□□□□□□□□$\neg\bar{\neg}(A \to B)$ (1)($Y_{\sim}$)

(6) □□□□$\bar{\neg} A$

(7) □□□□$A \to B$ (6)(MP5[1])

(8) □□□□$\neg(A \to B)$ (1)($Y_{\sim}$)

(9) □□□$\sim A$ (4)(5)(7)(8)(MP6[2])

Theorem 13 MP:

[1] If $A \vdash B, \sim A \vdash \sim B, \bar{\neg} A \vdash \bar{\neg} B$,

then $B \vdash A, \sim B \vdash \sim A, \bar{\neg} B \vdash \bar{\neg} A$.

[2] If $A \dashv\vdash B, \bar{\neg} A \dashv\vdash \bar{\neg} B$,

then $\sim A \dashv\vdash \sim B$

Proof of [1]: With the hypothesis we first prove $B \vdash A$.

(1) B

(2) □□□□□□□$\sim A$

(3) □□□□□□□$\sim B$ (2) hypothesis

(4) □□□□□□□$\neg B$ (3)($Y_{\sim}$)

(5) □□╕A (6) □□╕B (5) hypothesis

(7) □□$\neg B$ (6)($Y_{╕}$) (8) A (1)(4)(1)(7)(MP6[1])

Similarly, under the assumption we can use Theorem 6[2] and [3] to prove $\sim B \vdash \sim A$ ╕$B \vdash$ ╕A respectively.

Proof of [2]: Under the assumption we first prove $\sim A \vdash \sim B$, the proof of $\sim B \vdash \sim A$ is similar to this.

(1) $\sim A$

(2) □□□□□□B

(3) □□□□□□A (2)hypothesis

(4) □□□□□□$\neg A$ (1)($Y_{\sim}$)

(5) □□╕B

(6) □□╕A (5) hypothesis

(7) □□$\neg$╕A (1)($Y_{\sim}$)

(8) $\sim B$ (3)(4)(6)(7)(MP6[2])

Theorem 14 MP:

[1] $A \vdash \sim\sim A$

[2] ╕$A \vdash \sim\sim A$

Proof of [1]: By theorem 1[2] and ($\sim\sim$).

Proof of [2]: By theorem 5[1] and ($\sim\sim$).

Note that this theorem has already fixed the truth values of connective $\sim$ complelely i.e.

$A \vdash \sim(\sim A)$ means that $\sim A$ *fuz* when A is true,

╕$A \vdash \sim(\sim A)$ means that $\sim A$ *fuz* when A is false,

$\sim A \vdash \sim A$ means that $\sim A$ is true when A *fuz*.

Theorem 15 MP:

[1] $\vdash \neg$╕$\sim A$

[2] $\vdash \neg$╕$\neg A$

[3] ╕$\sim A \vdash B$

[4] ╕$\neg A \vdash B$

Proof of [1]:

(1) □□□□□□□$\sim A$

(2) □□□□□□□$\neg$╕$\sim A$ (1)(Y)

(3) □□□□□A

(4) □□□□□$\sim\sim A$ (3)(MP14[1])

(5) □□□□□$\neg$╕$\sim A$ (4)($Y_{\sim}$)

(6) □□□╕A

(7) □□□$\sim\sim A$ (6)(MP14[2])

(8) □□□$\neg$╕$\sim A$ (7)($Y_{\sim}$)

(9) $\neg$╕$\sim A$ (2)(5)(8)(MP11[5])

Proof of [2]:

(1) □□□□╕$\neg A$

(2) □□□□╕$(A \rightarrow \sim A)$ (1)($D(\neg)$)

(3) □□□□╕$\sim A$ (2)(╕$_{\rightarrow}$)

(4) □□□□$\neg$╕$\sim A$ (MP15[1])

(5) $\neg$╕$\neg A$ (3)(4)($\neg_{+}$)

Here we must point out that $A =_{df} B$ (i.e. A is definded as B) denotes that A is a different writing of B, and vice versa. So A (or B) in formulas can be substituted by B (or A). Then, from $D(\neg)$: $\neg A =_{df} A \rightarrow \sim A$. We can get ╕$\neg A \vdash$ ╕$(A \rightarrow \sim A)$. Besides, in putting $\neg$╕$\sim A$ straightly into the below of ╕$\sim A$ by Theorem 15[1], we in fact used the following rule of inference:

If $\vdash A$, then $\Delta \vdash A$ (Δ is non-empty),

which is the special case of (τ)(Δ is emply). In the method of inclied proof, the procedures of using the inference rules $(\in)$ and (τ) are usually omitted.

Proof of [3]: Using Theoren 15 [1] and Theorem 2[1].

Proof of [4]: Using Theorem 15[2] and Theorem 2[1].

Theorem 16 (Substituting theorem): If $A \vdash\dashv B$ and $\sim A \vdash\dashv B$ and $\neg A \vdash\dashv \neg B$, then for any well-formed formulas we have

$$f(A) \vdash\dashv f(B) \text{ and } \sim f(A) \vdash\dashv \sim f(B) \text{ and } \neg f(A) \vdash\dashv \neg f(B).$$

Proof According to the generating way of well-formed formulas of MP, the theorem can be proved by induction on the following four cases:

(i) If $f(P)$ is proposition variable P, then the conclusion of the theorem is just the premise of the theorem.

(ii) If $f(P)$ is a well-formed formula in the form of $\sim g(P)$, where $g(P)$ is a well-formed formula meeting the requirements of the theorem, we have:

(A) $g(A) \vdash\dashv g(B)$ and $\sim g(A) \vdash\dashv g(B)$ and $\neg g(A) \vdash\dashv \neg g(B)$, and $\sim g(A) \vdash\dashv \sim g(B)$ is just $f(A) \vdash\dashv f(B)$.

(B) It is easy to prove $\neg \sim g(A) \vdash \neg \sim g(B)$ by theorems 15[1] and 2[1], and vice versa. So

$$\neg \sim g(A) \vdash\dashv \neg \sim g(B)$$

and which implies $\neg f(A) \vdash\dashv \neg f(B)$.

(C) From (A)、(B) we have

$$f(A) \vdash\dashv f(B) \text{ and } \neg f(A) \vdash \neg f(B),$$

and then $\sim f(A) \vdash\dashv \sim f(B)$ can be derived directly by using theorem 13[2].

So, form (A)、(B)、(C) we know that when $f(P)$ is the well-formed formula in the form $\sim g(P)$, the theorem holds.

(iii) If $f(P)$ is a well-formed formula in the form of $\neg g(P)$, where $g(P)$ is a well-formed formula meeting the requirements of the therom, we have

(A) $g(A) \vdash\dashv g(B)$ and $\sim g(A) \vdash\dashv \sim g(B)$ and $\neg g(A) \vdash\dashv \neg g(B)$, and $\neg g(A) \vdash\dashv \neg g(B)$ is just $f(A) \vdash\dashv f(B)$.

(B) We first prove $\neg\neg g(A) \vdash \neg\neg g(B)$.

Proof (1) $\neg\neg g(A)$
(2) $g(A)$ — (1) $(\neg\neg_-)$
(3) $g(B)$ — (2) inductive hypothesis
(4) $\neg\neg g(B)$ — (3) $(\neg\neg_+)$

then $\neg\neg g(B)\vdash\neg\neg g(A)$ can be proved in the same way, so $\neg\neg g(A)\vdash\dashv\neg\neg g(B)$, and that is $\neg f(A)\vdash\dashv\neg f(B)$.

(C) From (A)、(B) and Theorem 13[2] we immediately have $\sim f(A)\vdash\dashv\sim f(B)$.

(A)、(B)、(C) imply that if $f(P)$ is a well-formed formula in the form of $\neg g(P)$, the theorem is also valid.

(iv) If $f(P)$ is a well-formed formula in the form of $g_1(P)\to g_2(P)$, where $g_1(P)$ and $g_2(P)$ are both well-formed formulas meeting the demands of the theorem,

(A) First, we prove $g_1(A)\to g_2(A)\vdash g_1(B)\to g_2(B)$

Proof (1) $g_1(A)\to g_2(A)$
(2) □□□□□□$g_1(B)$
(3) □□□□□□$g_1(A)$ — (2) inductive hypothesis
(4) □□□□□□$g_2(A)$ — (1)(3)$(\to_-)$
(5) □□□□□□$g_2(B)$ — (4) inductive hypothesis
(6) □□□$\sim g_1(B)$
(7) □□□$\sim g_1(A)$ — (6) inductive hypothesis
(8) □□□$g_2(A)$ — (1)(7)$(\to_-)$
(9) □□□$g_2(B)$ — (8) inductive hypothesis
(10) $g_1(B)\to g_2(B)$ — (5)(9)$(\to_+)$

then, $g_1(B)\to g_2(B)\vdash g_1(A)\to g_2(A)$ can be proved in the same way. So $g_1(A)\to g_2(A)\vdash\dashv g_1(B)\to g_2(B)$, which is just $f(A)\vdash\dashv f(B)$.

(B) We first prove $\neg(g_1(A)\to g_2(A)\vdash\neg(g_1(B)\to g_2(B))$

Proof: (1) $\neg(g_1(A)\to g_2(A))$
(2) $g_1(A)$ — (1)$(\neg_{\to})$
(2) $\neg g_2(A)$ — (1)$(\neg_{\to})$
(4) $g_1(B)$ — (2) inductive hypothesis
(5) $\neg g_2(B)$ — (3) inductive hypothesis
(6) $\neg(g_1(B)\to g_2(B))$ — (4)(5)$(\neg_{\to})$

$\neg(g_1(B)\to g_2(B))\vdash\neg(g_1(A)\to g_2(A))$ can be proved in the same way. So we have $\neg(g_1(A)\to g_2(A)),\vdash\dashv\neg(g_1(B)\to g_2(B))$, Which is just $\neg f(A)\vdash\dashv\neg f(B)$.

(C) From (A)、(B) and Theorem 13[2] we immediately have $\sim f(A)\vdash\dashv\sim f(B)$.

(A)、(B)、(C) imply that if $f(P)$ is a well-formed formula in the form of $g_1(P)\to g_2(P)$, the theorem is valid too. Q. E. D.

It should be pointed out that in the theorem, we have $f(A)\vdash\dashv f(B)$ for any well-formed formula $f(P)$. Since $\sim f(P)$ and $\neg f(P)$ are both well-formed formulas, we immediately have $\sim f(A)\vdash\dashv\sim f(B)$ and $\neg f(A)\vdash\dashv\neg f(B)$. But for the convenience and requirement in proof, we still describe the conclusion of the proof as: for any well-formed formula $f(P)$ we have

$$f(A)\vdash\dashv f(B) \text{ and } \sim f(A)\vdash\dashv\sim f(B) \text{ and } \neg f(A)\vdash\dashv\neg f(B).$$

It seems that the procedure of the proof has become more tedious and unnecessary owing the strenthening of conclusion, but in fact we have strenthened the induction at the same time of strenthening the conclusion, therefore the proof of $\sim f(A)\vdash\dashv\sim f(B$ and $\neg f(A)\vdash\dashv\neg f(B)$ is necessary.

Definition If for any well-formed formula $f(P)$ in MP we always have

$$f(A)\vdash\dashv f(B),$$

then A and B are called equality-valuations, in other words, any appearance of A and that of B in formula, can be substituted by each other, which is expressed as $A\models\dashv B$.

Here we must note that the symbol $\models\dashv$ is not a formal symbol of the vocabulary, but an abbreviation with some meaning, and we have only defined the meaning of $A\models\dashv B$, but not the notation of $\neg(A\models\dashv B)$ or $\sim(A\models\dashv B)$, thus these notations are all meaningless and we can not construct such expressions as $\vdash A\models\dashv B$.

Theorem 17 MP:

[1] If $A\vdash B$ and $\sim A\vdash\sim B$ and $\neg A\vdash\neg B$, then $A\models\dashv B$.

[2] If $A\vdash\dashv B$ and $\neg A\vdash\dashv\neg B$, then $A\models\dashv B$,

[3] If $\sim A\vdash\dashv\sim B$, then $\sim A\models\dashv\sim B$.

Proof of [1]: From the hypothesis of the theorem and theorem 13[1] we have

$$B\vdash A \text{ and } \sim B\vdash\sim A \text{ and } \neg B\vdash\neg A.$$

It meets the hypothesis of Theorem 16 (the Substitutive theorem), so we have $f(A)\vdash\dashv f(B)$ for any well-formed fomula $f(P)$, that is $A\models\dashv B$.

Proof of [2]: From the hypothesis of the theorem and theorem 13[2], we have $\sim A \dashv\vdash \sim B$, thus $A \dashv\vdash B$ and $╕ A \dashv\vdash ╕ B$ and $\sim A \dashv\vdash \sim B$. By the substitutive theorem we have $f(A) \dashv\vdash f(B)$ for any well-formed formula $f(P)$, that is $A \vDash\!\dashv B$.

Proof of [3]: First, we prove $╕\sim A \vdash ╕\sim B$.

(1) $╕\sim A$

(2) $\neg ╕ \sim A$ (MP15[1])

(3) $╕\sim B$ (1)(2)(MP2[1])

Then, similarly, we have $╕\sim B \vdash ╕\sim A$, thus with the hypothsis we have

$$\sim A \dashv\vdash \sim B \text{ and } ╕\sim A \dashv\vdash ╕\sim B \text{ ,}$$

Theorem 18 MP :

[1] $A \vDash\!\dashv ╕╕ A$; [2] $A \to A \vDash\!\dashv \sim\sim A$;

[3] $\sim ╕ A \vDash\!\dashv \sim A$.

Proof of [1]: From $(╕╕_{+})$ and $(╕╕_{-})$ we have $A \dashv\vdash ╕╕ A$, and similarly $╕ A \dashv\vdash ╕╕╕ A$. Then

$$A \dashv\vdash ╕╕ A \text{ and } ╕ A \dashv\vdash ╕╕╕ A,$$

so, with Theorem117[2], we learn $A \vDash\!\dashv ╕╕ A$.

Proof of [2]: First, we prove $╕(A \to A) \vdash ╕(\sim\sim A)$

(1) $╕(A \to A)$

(2) A (1)$(╕_{\to})$

(3) $╕ A$ (1)$(╕_{\to})$

(4) $\neg A$ (3)$(Y_{╕})$

(5) $╕\sim\sim A$ (2)(4)(MP2[1])

we have $╕\sim\sim A \vdash ╕(A \to A)$ by Theorems 15(1] and 2[1], so with the Rule $(\sim\sim)$ and by Theorem 17[2] we obtain $A \to A \vDash\!\dashv \sim\sim A$.

Proof of [3]: We have $\sim A \dashv\vdash \sim ╕ A$ by Theorems 4[1]、[2], and thus $\sim A \vDash\!\dashv \sim ╕ A$ is verified by Theorem 17[3].

Theorem 19 MP:

[1] $A \vdash \sim\neg A$; [2] $╕ A \vdash \sim\neg\neg A$; [3] $\sim A \vdash \sim\neg\neg A$.

Proof of [1]:

(1) A

(2) □□□□$\neg A$

(3) □□$╕\neg A$

(4) □□$\neg ╕ \neg A$ (MP15[4])

(5) $\vdash \sim\neg A$ (1)(2)(4)(MP6[2])

Proof of [2]:

(1) $╕ A$

(2) $\neg A$ (1)$(Y_{╕})$

(3) $\sim\neg\neg A$ (2)(MP19[1])

Proof of [3]:

(1) $\sim A$

(2) $\neg A$ (1)($Y_\sim$)

(3) $\sim\neg\neg A$ (2)(MP19[1])

It should be noted that by combination of Theorem 19[1] with axioms ($Y_\sim$) and ($Y_{\text{╕}}$) we have completely defined the truth values of $\neg A$, i.e.

$A \vdash \sim\neg A$ implies that $\neg A$ *fuz* when A is true,

$\text{╕}A \vdash \neg A$ means that $\neg A$ true when A is false,

$\sim A \vdash \neg A$ means that $\neg A$ is true when A *fuz*.

By the combination of Theorems 19[2],[3] with theorem 3[2], the truth values of $\neg\neg A$ have also be completely defined.

$A \vdash \neg\neg A$ means that $\neg\neg A$ is true when A is true.

$\sim A \vdash \sim\neg\neg A$ means that $\neg\neg A$ *fuz* when A *fuz*,

$\text{╕}A \vdash \sim\neg\neg A$ means that $\neg\neg A$ *fuz* when A is false.

from these we know that $A \vDash\!\dashv \neg\neg A$ is invalid because we do not have $\text{╕}A \vdash \text{╕}\neg\neg A$.

Theorem 20 MP:

[1] $\sim\sim\sim A \vDash\!\dashv \sim A$

[2] $\neg\neg\neg A \vDash\!\dashv \neg A$

Proof of [1]: First, we prove $\sim A \vdash \sim\sim\sim A$

(1) $\sim A$

(2) □□□□□□□$\sim\sim A$

(3) □□□□□□□$\neg\sim A$ (2)($Y_\sim$)

(4) □□$\text{╕}\sim\sim A$

(5) □□$\neg\text{╕}\sim\sim A$ (MP15[1])

(6) $\sim\sim\sim A$ (1)(3)(4)(5)(MP6[2])

Then we prove $\sim\sim\sim A \vdash \sim A$

(1) $\sim\sim\sim A$

(2) □□□□□□A

(3) □□□□□□$\sim\sim A$ (2)(MP14[1])

(4) □□□□□□$\neg\sim\sim A$ (1)($Y_\sim$)

(5) □□$\text{╕}A$

(6) □□$\sim\sim A$ (5)(MP14[2])

(7) □□$\neg\sim\sim A$ (1)($Y_\sim$)

(8) $\sim A$ (3)(4)(6)(7)(MP6[2])

so $\sim A \vdash\!\dashv \sim\sim\sim A$, and further by Theorem 17[3] we have $\sim A \vDash\!\dashv \sim\sim\sim A$.

Proof of [2]: First we prove $\neg\neg\neg A \vdash \neg A$

(1) $\neg\neg\neg A$

(2) □□□□A

(3) □□□□$\neg\neg A$ (2)(TH3[2])

(4) $\neg A$ (1)(3)($\neg_+$)

Then we prove $\neg A \vdash \neg\neg\neg A$

(1) $\neg A$

(2) □□□□$\neg\neg A$

(3) □□□□A (2)(TH3[1])

(4) $\neg\neg\neg A$ (3)(1)($\neg_{+}$)

It means $\neg A \vdash\dashv \neg\neg\neg A$

Now we prove $\text{╕}\neg\neg\neg A \vdash \text{╕}\neg A$

(1) $\text{╕}\neg\neg\neg A$

(2) $\neg\text{╕}\neg\neg\neg\neg A$ (MP15[2])

(3) $\text{╕}\neg A$ (1)(2)(MP2[1])

$\text{╕}\neg A \vdash \text{╕}\neg\neg\neg A$ can be proved in the same way, so $\text{╕}\neg A \vdash\dashv \text{╕}\neg\neg\neg A$. Hence by Theorem 17[2] we get $\neg A \models\!\dashv \neg\neg\neg A$.

This theorem says that if there are more than three defining symbols $\neg$ or connective symbol~before any well-formed formulas, these symbols or connectives may be cut down in even number while left at least one. But for connective symbol ╕there is no such a limitation as leaving at least one while it can be added in or cut down in even number at will.

Theorem 20 [1] and theorem 14 have completely defined the truth values of $\sim\sim A$, that is

$A \vdash \sim\sim A$ means that $\sim\sim A$ is true when A is true,

$\text{╕}A \vdash \sim\sim A$ means that $\sim\sim A$ is true when A is false,

$\sim A \vdash \sim\sim\sim A$ means that $\sim\sim A$ *fuz* when A *fuz*.

From the definition of the equality-valuations of A and B and the substitutive theorem we immediately verify the following theorem.

Theorem 21 If $A \models\!\dashv B$, then $f(A) \models\!\dashv f(B)$ for any well-formed formula $f(P)$.

Theorem 22 $A \rightarrow A \models\!\dashv \text{╕}B \rightarrow \text{╕}A$

Proof Let $*B$ be $\text{╕}B$ and $\varDelta B$ be B. By Theorems 7[7] and 9[3] we first have $A \rightarrow B \vdash\dashv \text{╕}B \rightarrow \text{╕}A$.

Now we prove $\text{╕}(A \rightarrow B) \vdash \text{╕}(\text{╕}B \rightarrow A)$

(1) $\text{╕}(A \rightarrow B)$

(2) □□□□□□□$\text{╕}B \rightarrow \text{╕}A$

(3) □□□□□□□□$A \rightarrow B$ (2)(MP9 3])

(4) □□□□□□□□$\neg(A \rightarrow B)$ (1)($Y_{\text{╕}}$)

(5) □□□$\sim(\text{╕}B \rightarrow \text{╕}A)$

(6) □□□$\text{╕}B$ (1)($\text{╕}_{\rightarrow}$)

(7) □□□$\sim\text{╕}A$ (5)(6)(MP12[4])

(8) □□□A (1)($\text{╕}_{\rightarrow}$)

(9) □□□$\text{╕}\text{╕}A$ (8)($\text{╕}\text{╕}_{+}$)

(10) □□□$\neg\sim\text{╕}A$ (9)($Y_{\text{╕}}$)

(11) ╕(╕B → ╕A)   (3)(4)(7)(10)(MP6[3])

Then we prove ╕(╕B → ╕A) ⊢ ╕(A → B)

(1) ╕(╕B → ╕A)

(2) □□□□□□□ A → B

(3) □□□□□□□ ╕B → ╕A (2)(MP7[7])

(4) □□□□□□□ ¬(╕B → ╕A)   (1)($Y_{╕}$)

(5) □□□□ ~(A → B)

(6) □□□□ ╕ ╕A (1)($╕_{\rightarrow}$)

(7) □□□□ A (6)($╕╕_{-}$)

(8) □□□□ ~B (5)(7)(MP112[4])

(9) □□□□ ¬╕B (8)($Y_{\sim}$)

(10) □□□□ ╕B (1)($╕_{\rightarrow}$)

(11) ╕(A → B)   (3)(4)(9)(10)(MP6[3])

Then we have also ╕(A → B) ⊣⊢ ╕(B B → ╕A). Therefore from Theorem 17[2] we have A → B ⊨⫤ ╕B → ╕A.

References

[1] Zhu Wujia, Xiao Xian, Nature Journal V. 7, No. 10(1984).

[2] Zhu Wujia, Xiao Xian, On the Naive Mathematical Models of Medium Mathematical System MM, JMRE. 8(1), 1988, 1139-151.

[3] Hu Shihua, Liu Zhongwan, Foundation of Mathematical Logic, Science Press, 1981.

[4] Xiao Xian, Zhu Wujia, Propositional Calculus System of Medium Logic (I), JMRE, 8(2), 1988.