Journal of Mathematical Research and Exposition
Vol.9, No.1, Feb.1989

Complete Convergence for ρ-mixing Sequences *

Shao Qiman

(Dept. Math., Hangzhou University)

Let $\{X_k, k\geqslant 1\}$ be a sequence of random variables and let $S_n=\sum_{k=1}^{n} X_k$. Denote $F_n^k=\sigma(X_i, n\leqslant i\leqslant k)$. The sequence $\{X_k\}$ is said to be ρ-mixing if

$$\rho(n)=\sup_k \sup_{X\in L_2(F_1^n),\ Y\in L_2(F_{k+n}^\infty)} \frac{|EXY-EXEY|}{(\mathrm{Var}X\ \mathrm{Var}Y)^{1/2}}\to 0\ (n\to\infty).$$

Let $l(x)$ and $\beta(x)$ be even positive functions. Suppose there is a censtant $\theta>0$ such that $\beta(x)/x^\theta$, $x^2/\beta(x)$ and $l(x)$ $(x>0)$ are monotone nondecreasing Denote $a(x)$ is the converse funotion of $\beta(x)$.

Theerem 1 Suppose there is a constant $0<\varepsilon_0<1$ such that $\beta(x)l(\beta(x))/x$ and $x^{2-\varepsilon_0}/(\beta(x)l(\beta(x)))$ are monotone nondecreasing. Let $\{X_k\}$ be a ρ-mixing sequence of identically distributed random varibles with $EX_1=0$ and $E\beta(X_1)l(\beta(|X_1|))<\infty$. Suppose that $\sum_{n=1}^{\infty}\rho(2^n)<\infty$. Then

$$\sum_{n=1}^{\infty}\frac{l(n)}{n}P(\max_{i\geq n}\beta(S_i)\geqslant\varepsilon n)<\infty, \qquad \forall\,\varepsilon>0 \tag{1}$$

Theerem 2 Suppose there are constants $q\geqslant 2$, $q_0\geqslant 2$ and $\delta>0$ such that $x^q/(\beta(x)l(\beta(x)))$ and $\beta(x)l(\beta(x))/x^2$ are monotone nondecreasing, $l(x)\geqslant x^\delta$ $(x>0)$ and $\sum_{n=1}^{\infty}\frac{l(n)}{n}\left(\frac{n^{1/2}}{a(n)}\right)^{q_0}<\infty$.
Let $\{X_k\}$ be a ρ-mixing sequence of identically distributed random variables with $EX_1=0$ and $E\beta(X_1)l(\beta(X_1))<\infty$. Suppose there is a consthant $r>q$ such that $\sum_{n=1}^{\infty}\rho^{2/r}(2^n)<\infty$. Then (1) holds.

Corollary Let $\{X_k\}$ be a ρ-mixing sequence of identically distributed random variables with $EX_1=0$ and $E|X_1|^p\log(1+|X_1|)<\infty$ for $1\leqslant p<2$. Suppose that $\sum_{n=1}^{\infty}\rho(2^n)<\infty$. Then $\sum_{n=1}^{\infty}\frac{\log n}{n}P(\max_{i\leq n}|S_i|\geqslant\varepsilon n^{1/q})<\infty$. $\forall\,\varepsilon>0$

This is the result desired by Su Chun(1986).

* *Received March* 25, 1987.