Journal of Mathematical Research and Exposition
Vol.9, No.4, Nov., 1989

Dominance Theory and Plane Partitions*

IV. Enumeration of Row and Column-Strict Reverse Plane Partitions

Chu Wenchang

(Research Institute of Applied Mathematics, DIT)

Let $RG_\lambda(\overline{n};q)$ denote the GF for row and column-strict reverse plane partitions of shape λ with part restriction $\overline{n}$ (i.e., the parts in ith row do not exceed n_i). Using the correspondence similar to that in section III with the same title, we have the following determinant expression.

Theorem 4.1.

$$RG_\lambda(\overline{n};q)=q^{n(\lambda)+n(\lambda')}\det_{k\times k}\left\|\begin{bmatrix}n-i+2\\ j-i+\lambda_i\end{bmatrix}q^{\binom{j-i}{2}+(j-i)\lambda_i}\right\|$$

The simplified forms are as follows:

Theorem 4.2.

$$RG_\lambda(n\overline{I}+\overline{\lambda};q)=q^{n(\lambda)+n(\lambda')}\prod_{(i,j)\in\lambda}\frac{\langle n+c_{ij}+2\rangle}{\langle h_{ij}\rangle}.$$

Theorem 4.3.

$$RG_\lambda(n\overline{I}+\overline{J};q)=q^{n(\lambda)+n(\lambda')}\prod_{(i,j)\in\lambda}\frac{\langle n-c_{ij}+2\rangle}{\langle h_{ij}\rangle}.$$

Corollary 4.4

$$RG_{r*c}(n;q)=q^{\binom{r}{2}c+\binom{c}{2}r}\prod_{i=1}^{r}\begin{bmatrix}n-i+2\\ n-r-c+2\end{bmatrix}\Big/\begin{bmatrix}n-c-i+2\\ n-r-c+2\end{bmatrix}.$$

And dually

$$RG_{r*c}(n;q)=q^{\binom{r}{2}c+\binom{c}{2}r}\prod_{j=1}^{c}\begin{bmatrix}n-j+2\\ n-r-c+2\end{bmatrix}\Big/\begin{bmatrix}n-r-j+2\\ n-r-c+2\end{bmatrix}$$

Corollary 4.5. (Stanley, 1971).

$$RG_\lambda(\infty;q)=q^{n(\lambda)+n(\lambda')}\prod_{(i,j)\in\lambda}\langle h_{ij}\rangle^{-1}.$$

* Received Jun. 27, 1987