Journal of Mathematical Research and Exposition
Vol.10 No.1 Feb. 1990

Dominance Theory and Plane Partitions*

VI. Enumeration of Column-Strict Plane Partitions

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Let $F_\lambda^{(\lambda)}(\overline{n};q)$ denote the GF for column strict plane partitions of shape λ with part restrictions $\overline{n}$ and n (i.e., the parts in ith row are not less than n_i and the largest parts in partitions do not exceed n). Using the correspondence similar to that in section V with the same title, we get the following determinant expression.

Theorem 6.1

$$F_\lambda^{(n)}(\overline{n};q)=q^{\langle\overline{n},\overline{\lambda}\rangle}\det_{k\times k}\left[\begin{bmatrix} n-n_i-i+\lambda_i+1 \\ j-i+\lambda_i\end{bmatrix} q^{(j-i)n_i}\right].$$

The simplified results are as follows:

Theorem 6.2

$$F_\lambda^{(n)}(\overline{\lambda}-\overline{J};q)=q^{2n(\lambda')-n(\lambda)}\prod_{(i,j)\in\lambda}\frac{\langle n-c_{ij}+1\rangle}{\langle h_{ij}\rangle}$$

Theorem 6.3 (Macdonald, 1979)

$$F_\lambda^{(n)}(0;q)=q^{n(\lambda)}\prod_{(i,j)\in\lambda}\frac{\langle n+c_{ij}+1\rangle}{\langle h_{ij}\rangle}$$

Corollary 6.4 (Gordon & Houten, 1968)

$$F_\lambda^{(\infty)}(0;q)=q^{n(\lambda)}\prod_{(i,j)\in\lambda}\langle h_{ij}\rangle^{-1}.$$

Corollary 6.5

$$F_{r\cdot c}^{(n)}(0;q)=q^{\binom{r}{2}c}\prod_{i=1}^{r}\begin{bmatrix} n+c-i+1 \\ c\end{bmatrix}\Big/\begin{bmatrix} c+i-1 \\ c\end{bmatrix},$$

and dually

$$F_{r\cdot c}^{(n)}(0;q)=q^{\binom{r}{2}c}\prod_{j=1}^{c}\begin{bmatrix} n+r-j+1 \\ r\end{bmatrix}\Big/\begin{bmatrix} r+j-1 \\ r\end{bmatrix}$$

* Received Jun. 27, 1987.