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Dominance Theory and Plane Partitions*

VII. Enumeration of Row and Column Stirct Plane Partitions

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Let $G_\lambda^{(n)}(\overline{n}; q)$ denote the GF for row and column strict plane partitions of shape λ with part restrictions $\overline{n}$ and n (i.e., the parts in ith row are not less than n_i and the largest parts in partitions do not exceed n). Using the correspondence similar to that in section V with the same title, we establish the following determinant expression.

Theorem 7.1

$$G_\lambda^{(n)}(\overline{n}; q) = q^{n(\lambda')+\langle \overline{n}, \overline{\lambda}\rangle}\det_{k\times k}\left[\begin{bmatrix} n-n_i-i-2 \\ j-i+\lambda_i\end{bmatrix} q^{(j-i)(n_i+\lambda_i)}\right]$$

The following are its simplified forms.

Theorem 7.2

$$G_\lambda^{(n)}((m+2)\overline{I}-\overline{\lambda}; q) = q^{(m+1)|\overline{\lambda}|+n(\lambda)-n(\lambda')}\prod_{(i,j)\in\lambda}\frac{\langle n+c_{ij}-m\rangle}{\langle h_{ij}\rangle}$$

Theorem 7.3

$$G_\lambda^{(n)}((m+2)\overline{I}-\overline{J}; q) = q^{(m+1)|\overline{\lambda}|-n(\lambda)+n(\lambda')}\prod_{(i,j)\in\lambda}\frac{\langle n-c_{ij}-m\rangle}{\langle h_{ij}\rangle}$$

Corollary 7.4

$$G_{r*c}^{(n)}(0; q) = q^{\binom{r}{2}c+\binom{c}{2}r}\prod_{i=1}^{r}\begin{bmatrix} n-i+2 \\ c\end{bmatrix}\Big/\begin{bmatrix} c+i-1 \\ c\end{bmatrix}.$$

And dually

$$G_{r*c}^{(n)}(0; q) = q^{\binom{r}{2}c+\binom{c}{2}r}\prod_{j=1}^{c}\begin{bmatrix} n-j+2 \\ r\end{bmatrix}\Big/\begin{bmatrix} r+j-1 \\ r\end{bmatrix}.$$

Corollary 7.5.

$$G_{r*c}^{(\infty)}(0; q) = q^{\binom{r}{2}c+\binom{c}{2}r}\prod_{i=1}^{r}\prod_{j=1}^{c}\langle i+j-1\rangle^{-1}.$$

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