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The Approximation of Continuous Functions by Jackson Operator

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Let $f(x): R' \to R'$ be a continuous function with period 2π, i.e, $f(x) \in C_{2\pi}$. D. Jackson singular integral operator of $f(x)$ is defined by:

$$J_n(f, x) = \frac{3}{2n\pi(2n^2+1)} \int_{-\pi}^{\pi} f(t) \left[\frac{\sin n\frac{t-x}{2}}{\sin\frac{t-x}{2}} \right]^4 dt \qquad (1)$$

According to [1], for any $f(x) \in C_{2\pi}$, Then

$$|J_n(f, x) - f(x)| \leqslant 6\omega(f, \frac{1}{n}),$$

where $\omega(f, \cdot)$ is the modulus of continuity of function $f(x)$.

Lemma $\int_0^{\frac{\pi}{2}} t\left(\frac{\sin nt}{\sin t}\right)^4 dt \leqslant \left(\frac{n^2}{9} + \frac{89}{288}\right)\pi^2$ for $n > 2$.

By virtue of Lemma, we show that (2) can be improved as the following result:

Theorem For any $f(x) \in C_{2\pi}$, the following approximation holds when $n \geqslant 50$.

$$|J_n(f, x) - f(x)| \leqslant \left[\pi\left(\frac{2}{3} + \frac{267}{360000}\right) + 1\right]\omega(f, \frac{1}{n}) = 3.096725115\omega(f, \frac{1}{n})$$

References

[1] Natanson, I.P., The constructive theory of functions, Moscow, 1949.

[2] Wang, R.H, Approximation of Unbounded Functions, Science Publishing House, China, 1983. In chinese.

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