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Contributions to Connected Domination in Graphs*

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Let G be a simple graph with n vertices. Let $D \subset V(G)$. If for any $v \in V(G)-D$, there exists $u \in D$ such that $uv \in E(G)$, then D is said a domination set of G. Let D be a domination set of G. If the induced subgraph $\langle D \rangle$ is connected (independent, resp.), then D is said a connected domination set (independent domination set, resp.). For any $v \in V(G)$, the closed neighborhood of v is defined as: $N[v] = \{u \mid v = v \text{ or } uv \in E(G)\}$. Let $X \subset V(G)$. If for any $v \in X$, $N[v] \not\subset U_{u \in X-\{v\}} N[u]$, then X is said an irredundance set. Let

$\gamma(G) = \min\{|D| \,|\, D \subset V(G)$ is a domination set of $G\}$;

$i(G) = \min\{|D| \,|\, D \subset V(G)$ is an independent domination set of $G\}$;

$\gamma_c(G) = \min\{|D| \,|\, D \subset V(G)$ is a connected domination set of $G\}$;

$ir(G) = \min\{|x| \,|\, X \subset V(G)$ is a maximal irredundance set of $G\}$;

$d_c(G)$ denotes the largest order of a partition of $V(G)$ into connected domination sets;

$\kappa(G)$ denotes the connectivity of G.

In this paper, the following theorems are proved.

Theorem 1 If G is a connected graph with $n > 3$, then $i(G) + \gamma_c(G) < n + \lceil n/3 \rceil - 2$.

Corollary 1 If G is a connected graph with $n > 3$, then $\gamma(G) + \gamma_c(G) < n + \lceil n/3 \rceil - 2$.

Theorem 2 If G is a connected graph, then $\gamma_c(G) < 4ir(G) - 2$.

Theorem 3 If both G and $\overline{G}$ are connected, then $\gamma_c(\overline{G}) < \kappa(G) + 1$.

Theorem 4 If G is a connected graph, and $G \neq K_n$, then $d_c(G) < \kappa(G)$.

Theorem 1 solves a problem of S. T. Hedetniemi and R. Laskar. The following open problems are posed in the paper.

Problem 1 Characterize the class of graphs satisfying $i(G) + \gamma_c(G) = n + \lceil n/3 \rceil - 2$.

Problem 2 Characterize the class of graphs satisfying $\gamma(G) + \gamma_c(G) = n + \lceil n/3 \rceil - 2$.

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Conjecture 3 If G is a connected graph, then $\gamma_c(G) \leqslant 3ir(G) - 2$.

Conjecture 4 If both G and $\overline{G}$ are connected, then for any real number $c > 0$, there exist n-order graphs G such that $\gamma_c(G)\gamma_c(\overline{G}) > cn$.

Conjecture 5 If both G and $\overline{G}$ are connected, then $\gamma_e(G) \leqslant 3d_c(\overline{G})$.

简　　讯

英文版数学杂志《逼近论及其应用》(Approximation Theory and its Applications)国内正副主编会议于1989年10月26日至28日在南京召开。

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会议对进一步提高刊物质量和印刷水平、扩大国内国际影响以及经费问题提出了一些积极的措施。会议吸收南京师范大学为新的资助单位。

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