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The Conditions for the Existance of n Limit-Cycles in Lienard Equation*

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The Lienard equation is

$$\ddot{x} + f(x)\dot{x} + g(x) = 0$$

For this equation we assume

1) The function $f(x)$ and $g(x)$ are continuous on $[\overline{a}_n, a_n]$, where $\overline{a}_n \cdot a < 0$.

2) There is a sequance

$$\overline{a}_n < \overline{a}_{n-1} < \cdots < \overline{a}_1 < 0 < a_1 < \cdots < a_{n-1} < a_n$$

satisying that $f(\overline{a}_i) = f(a_i) = 0$ $(i = 1, 2, \cdots, n)$. And we have

$$f(x) < 0 \quad \text{for} \quad x \in (\overline{a}_1, a_1),$$

$$f(x_i) \cdot f(x_{i+1}) < 0 \quad \text{for} \quad x_i \in (a_i, a_{i+1}), \quad x_{i+1} \in (a_{i+1}, a_{i+2}),$$

$$f(\overline{x}_i) \cdot f(\overline{x}_{i+1}) < 0 \quad \text{for} \quad \overline{x}_i \in (\overline{a}_{i+1}, \overline{a}_i), \quad \overline{x}_{i+1} \in (\overline{a}_{i+2}, \overline{a}_{i+1}),$$

where $i = 0, 1, 2, \cdots, n-2$; and $\overline{a_0} = a_0 = 0$ (Take the similar sign in the following conditions).

3) $xg(x) = 0$ $(x \neq 0)$ and $g(0) = 0$.

We take the symbols:

$$F(x) = \int_0^x f(x)\,dx, \quad G(x) = \int_0^x g(x)\,dx,$$

$$\Delta F_i = F(a_i) - F(a_{i-1}), \quad \Delta\overline{F_i} = F(\overline{a}_i) - F(\overline{a}_{i-1}),$$

$$\Delta G_i = G(a_i) - G(a_{i-1}), \quad \Delta\overline{G}_i = G(\overline{a}_i) - G(\overline{a}_{i-1}),$$

$$M_n = \max_{\substack{1 < i < n \\ 1 < j < n+1}} \{|\Delta F_i|, |\Delta\overline{F}_i|, 2[\Delta G_j]^{\frac{1}{2}}, 2[\Delta\overline{G}_i]^{\frac{1}{2}}\},$$

$$M_{G,n} = \max_{1 < i < n+1} \{2[\Delta G_i]^{\frac{1}{2}}, 2[\Delta\overline{G}_i]\},$$

$$m_{F,n} = \min_{1 < i < n} \{|\Delta F_i|, |\Delta\overline{F}_i|\},$$

$$K_2 = (1 + \sqrt{2}), \ K_4 = [1 + (K_2^2 + 2)^{\frac{1}{2}}], \cdots, K_{2r} \quad [1 + (K_{2r-2}^2) + 2)^{\frac{1}{2}}],$$

4) $\min\{F^2(a_{2r+1}), F^2(\overline{a}_{2r+1})\} \geq M_{G,2r}^2$.

5) $m_{F,n} > 2M_{G,n}$.

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关于部分变元Lyapunov矩阵方程几个问题的讨论

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摘　　要

关于Lyapunov矩阵方程 $A^TB+BA=-C$的解与线性定常系统 $\dot{x}=Ax$之零解的部分变元渐近稳定性的关系，本文就最近发表的一些结果讨论了如下几个问题．一、由于全变元正定函数也满足部分正定性的条件，有必要引进严格部分正定函数的定义．严格部分正定函数与全变元正定函数是互不相包的；二、求解矩阵方程即意味着对于给定的矩连A(它使系统$\dot{x}=Ax$之零解对部分变元渐近稳定)，矩阵C应满足什么条件使矩阵方程有解B，此即 Lyapunov函数的存在与构造问题；本文还指出C的秩不必为m，当 rank$C>m$时矩阵方程可能有解或无解；又（A^TB+BA)的秩不一定等于$m+h$；三、对于满足有关定理条件的矩阵A、C，矩阵方程 $A^TB+BA=-C$的解B中不仅有部分正定的，亦可能有正定的、不定的等，而其中部分正定的解矩阵B的唯一性并不成立．

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6) $\min\{F^2(a_{2r+1}),\ F^2(\overline{a_{2r+1}})\}\geqslant[(K_{2r}M_{2r}-m_{F,2r})^2+M^2_{G,2r}]$.

7) $F(a_{2r})>K_{2r}M_{2r-1}$, $F^2(\overline{a_{2r+1}})<-K_{2r}M_{2r-1}$.

where 4), 6), 7) are holding for $r=1,2,\cdots,k$.

In this paper, at first we construct eight lemmas, secondly we deduce six estimating expressions for the values of the path-curves of the Lienard equation, last we obtain the following result.

Theorem If the Lienard equation, satisfies the conditions from 1) to 7) as stated above, then it has $n=2k$ limit-cycles at least. If it satisfies the conditions from 1) to 7), and 7) is holding for $r=k+1$; then it has $n=2k+1$ limit-cycles at least.

Example Let $f(x)=-(x^2-1)(x^2-3^2)$ and $g(x)=\frac{1}{2}x$ in Lienard equation, then it has two limit-cycles on the interval $[-5,5]$ at least.

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