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Boundability of Open Manifolds*

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An open manifold in question is assumed to be a noncompact manifold with compact boundary (probably empty). Let M be a connected open n-dimensional smooth manifold. Take the one-point-compactification of M, it will be denoted by $\overline{M}=M\cup\{*\}$, where $*$ is the infinite point. Since M has countable basis, $\overline{M}$ is metrizable and $\{*\}$ is a closed subset and a G_δ set. By Urysohn lemma, there exists a continuous function $f:\overline{M}\to[0,1]$ with $f^{-1}(0)=\partial M$ and $f^{-1}(1)=*$. Choose a suitable approximation we may get a continuous function $\overline{f}:\overline{M}\to[0,1]$ such that $\overline{f}/M$ is smooth, $\overline{f}^{-1}(0)=\partial M$, $\overline{f}^{-1}(1)=*$, $\overline{f}$ has no degenerate critical points over M and then has at most countably many critical points. Such a function $\overline{f}$ is called a Morse function on $\overline{M}$. The Morse number $\mu(M)$ of M is the minimum over all Morse functions f on $\overline{M}$ of the number of critical points of f. If $\mu(M)$ is finite, M is called to be of finite type; otherwise M is called to be of infinite type.

An open smooth n-manifold M is called to be boundable, if there exists a compact smoth n-manifold N and a smooth imbedding $i:M\hookrightarrow N$ such that $N\setminus i(M)\subset\partial N$. Such manifold N is unique up to h-cobordism and is called a bounded manifold of M.

Theorem A connected open smooth manifold is boundable if and only if it is of finite type.

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