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Asymptotic Normality of Kernel Density Estimates*

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Let $X_1, \cdots, X_n$ be a random samples taking values in $\mathbf{R}^1$ and having common density function $f(x)$. A class of estimators of $f(x)$ proposed by Rosenblatt[1] has the form

$$f_n(x) = \frac{1}{nh_n} \sum_{i=1}^{n} K\left(\frac{x - X_i}{h_n}\right)$$

where K is a probability density function on R^1 and $\{h_n\}$ is a sequence of positive numbers converging to zero. Call $f_n(x)$ the kernel estimator of $f(x)$. Many autors studied the large sample properties of $f_n(x)$ under the independent samples. For examples: Parzen[2] and Sun[3] showed the asymptotic normality of $(f_n(x) - Ef_n(x))/\sqrt{\mathrm{Var} f_n(x)}$ respectively. Our paper aims to show the asymptotic normality of joint distribution of $\sqrt{nh_n}(f_n(x_1) - f(x_1), \cdots, f_n(x_k) - f(x_k))$ under the condition that samples $\{X_n, n \geqslant 1\}$ is a sequence of stationary、φ-Mixing variables. Our main result is

Theorem Suppose that (Ⅰ) $\sum_{n=1}^{\infty} \varphi(n) < \infty$; (Ⅱ) $(1 + |x|) K(x) < C < \infty$; $\int |y| K(y) \mathrm{d}y < \infty$; (Ⅲ) $nh_n \to \infty$, $nh_n^3 \to 0$, as $n \to \infty$; (Ⅳ) $f(x)$ satisfies Lipschitz condition and joint density $g_i(y_1, y_2)$ of (X_1, X_i) is a continuous bounded function for any $i > 1$. Then $\sqrt{nh_n}(f_n(x_1) - f(x_1), \cdots, f_n(x_k) - f(x_k))$ converges in distribution to Z for the distinct point $(x_1, \cdots, x_k)$ where $f(x_i) > 0$ $(i = 1, \cdots, k)$. Where Z is a k-dimensional normal vector and having mean vector 0 and diagonal covariance matric $B = (b_{ij})$, $b_{ii} = f(x_i) \int K^2(y) \mathrm{d}y$, $i = 1, \cdots, k$.

References

[1] Rosenblatt, M., Ann. Math. Statist., 27(1956), 832—837.
[2] Parzen, E., AMS, 33(1962), 1065—1076.
[3] Sun Zhigang, Acta. Math. Sinica, 27(1984), 769—782.

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