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Semi-Group Rings of Ordered Semi-Groups Which Are Reduced-Rings

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Throughout this note S will be an ordered semi-group with identity and R a commutative ring. RS the semi group ring of S over R. A ring R is said to be a reduced ring if R has no nonzero nilpotent elements. We say a semi-group $S(\neq 1)$ is ordered if it admits a linear ordering $<$, such that $g<h$ implies $gk<hk$, $kg<kh$ for all k in S (refer [1]).

We prove the following lemma to prove our main theorem.

Lemma 1 Let R be a reduced ring and $S(\neq 1)$ be an ordered semi group then the semi-group ring RS is a reduced ring.

Proof To show $x^n=0$ is not possible for any $x\neq 0$ in RS, n a positive integer. Let $x=\sum_{i=1}^{n} x_i s_i$. Given S is an ordered semi-group hence let $s_1<s_2<s_3<\cdots<s_n$, Consider $x^n=\sum_{i=1}^{n} s_i^n s_i^n+$ (terms as products of $s_i s_j$'s taken n at a time). Given $x_i^n\neq 0$ for $i=1,2,\cdots,m$ and we have s_i^n to be the largest element in the product so $x^n\neq 0$. Hence RS is a reduced ring.

The following example throws some light on the converse part of the above lemma which cannot be true if S is not ordered.

Example Let R be a commutative ring with identity. Let R be a reduced ring and S a semi-group commutative but non cancellative in which

(1) $s^3=t^3$ for every s and t in S.

(2) $s^2t=st^2$ for every s and t in S.

Then the semi-group ring RS is not a reduced ring. For take $x=s-t$ then $x^3=s^3-3s^2t+3st^2-t^3$, thus $x^3=0$.

So RS is not a reduced ring. Hence we impose the condition on S to be ordered.

Theorem 2 Let S be an ordred semi-group. The semi-group ring RS is a reduced ring if and only if R is a reduced ring. (to: 493)

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$\widetilde{G}_2 \cup \widetilde{G}_2$ in (R_3)

$(c)'$: $$(b \cdot v_1) \cdot H \geqq 0 \text{ on } N\mathrm{ch}(D')$$

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from: 494

Proof If RS is a reduced ring then $R \subseteq RS$ is a reduced ring.

Clearly if R is a reduced ring and S is an ordered semigroup by lemma 1 RS is a reduced ring.

Here it is interesting to note if we relax the condition that S need not be ordered then by above example we cannot always assert that RS to be a reduced ring.

But we pose the following problem.

Problem Can Theorem 2 be true for semi-groups which cannot be ordered?

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