Journal of Mathematical Research and Exposition
Vol. 10, No.4, Nov., 1990

Fixed Point Theorems for Contraction Mapping on Probabilistic Metric Spaces*

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Let R denote the reals, (E, F, Δ) be a Menger space. For any pair $p, q \in E$, $t \in R$, $F(p, q, t)$ is a distribution function on R.

A function $\phi(t): R_+ \to R_+$ is said to satisfy the condition (ϕ), if $\phi(0)=0$ and $\lim\limits_{n\to\infty}\phi^n(t)=\infty$, $\forall t>0$, where $\phi^n(t)$ is a n-th iteration of $\phi(t)$.

Throughout of this paper, we always assume that (E, F, Δ) is a complete Menger space, Δ is a continuous t-norm, $\phi(t)$ is a function satisiying the condition (ϕ).

Theorem 1 Let T be a continuous self-mapping on (E, F, Δ), A is a subset of E, $B=\bigcup\limits_{n=0}^{\infty} A_n$, $B_n=\bigcup\limits_{j=n}^{\infty} A_j$, $A_n=T^nA$, $n=0,1,2,\cdots$ If B is probabilistically bounded and for any B_m, there is a positive integer $m(B_m)$, $m=0,1,2,\cdots$ such that

$$\inf_{p,q\in T^{m(B_m)}B_m} F(p,q,t) \geqslant \inf_{p,q\in B_m} F(p,q,\phi(t))$$

Then T has a unique fixed point $p^*\in E$ such that for any $p_n \in T^nA$, $P_n \to p^*$.

Corollary 1 Let T be a continuous self-mapping on $(E, F\Delta)$, B_n, B as same as those in theorem 1. If B is probabilistically bounded, and there is a positive integer m such that for all $n>0$.

$$\inf_{p,q\in T^mB_n} F(p,q,t) \geqslant \inf_{p,q\in B_n} F(p,q,\phi(t))$$

Then the conclusion of theorem 1 still holds.

Corollary 2 Let (E, F, Δ) be a complete Menger space, Δ a t-norm satisfuing the condition $\Delta(t,t)\geqslant t$, $t\in[0,1]$. Suppose that $T: E\to E$ is a contraction mapping, i.e. there is a constent $k\in(0,1)$ such that

$$F(T_p, T_q, t) \geqslant F(p, q, \frac{t}{k}) \quad \forall t>0, \quad p, q\in E$$

Then T has a unique fixed point $p^*\in E$, and for each $p_0\in E$, the sequence $\{p_n=T^np_0\}$ converges to p^*.

Theorem 2 Let T be a continuous self-mapping on (E, F, Δ), sets B_m, A_n

* Received May. 11, 1987.

as same as those in theorem 1, B probabilistically bounded. For any A_j, B_k, j, $k=0,1,2,\cdots$ there exist positive integers $n(A_j)$, $m(B_k)$ such that

$$\inf_{\substack{p\in T^{n(A_j)}A_j \\ q\in T^{m(B_k)}B_k}} F(p,q,t) > \inf_{\substack{p\in A_j \\ q\in B_k\cup T^{n(A_j)}A_j}} F(p,q,\phi(t)) \tag{4}$$

Then the conclusion of theorem 1 still holds.

Theorem 3 Let T be a continuous self-mapping on (E, F, Δ), sets B_m, A_n as same as those in theorem 1. If B is probabilistically bounded, and for any A_n, there is a positive integer $m(A_n)$ such that

$$\inf_{\substack{p\in T^{m(A_n)}A_n \\ q\in T^{m(A_n)}B_m}} F(p,q,t) > \inf_{\substack{p\in A_n \\ q\in B_m\cup T^{m(A_n)}A_n}} F(p,q,\phi(t))$$

Then the conclusion of theorem 1 still holds.

Corollary 3 Let T be a self-mapping on (E, F, Δ), suppose that for each $x\in E$, the sequence $\{T^n x = x_n\}$ is probabilistically bounded, and for each $x\in E$, there exists a integer $m(x)\geq 1$ such that for all $y\in E$, $t>0$.

$$F(T^{m(x)}x, T^{m(x)}y, t) \geq \min\{F(x, y, \frac{t}{k}), F(x, T^{m(x)}y, \frac{t}{k}), F(x, T^{m(x)}x, \frac{t}{k})\}$$

where k is a constent with $k\in(0,1)$. Then T has a unique fixed point x^* and $T^n x \to x^*$ for each $x\in E$.

References

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概率度量空间中压缩映象的不动点定理

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本文通过引入压缩映象集序列的概念，推广并综合了张石生[1]以及V. M. Sehger和A. T. Bharucha-Reid[2]中某些主要定理的结果.