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热传导方程的移动边界问题*

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摘 要

热传导问题于高温条件下，往往是可移动边界问题．文献［1］尽述了金属丝烧蚀等物理过程所确定的移动边界问题的一种求解方法．本文讨论较一般的热传导方程可移动边界问题Fourier型存在的充分必要条件，且给出问题Fourier型解．

一、移动边界的确定

设问题

$$
(\mathrm{A})\left\{\begin{array}{ll}
u_t=a^2u_{xx}+\dfrac{\gamma x+\delta}{a+\beta t}u_x-\dfrac{ex^2+hx+f}{(a+\beta t)^2}, & (x,t)\in D=\{(x,t):0<x<l(t),\ 0<t<T\} \quad (1.1)\\
u(0,t)=u(l(t),t)=0 & (1.2)\\
u(x,0)=F(x),\ 0\le x\le l_0 & (1.3)\\
F(0)=F(l_0)=0. & (1.4)
\end{array}\right.
$$

其中$a\in R$, $a,\beta,\gamma,e,f\in R^+$, $\delta\le 0$, $h=\dfrac{\delta\beta}{2a^2}(1+\dfrac{\gamma}{\beta})$，且有Fourier型形式解．

$$u(x,t)=\sum_{n=1}^{\infty}[A_n\sin\frac{n\pi x}{l(t)}+B_n\cos\frac{n\pi x}{l(t)}]e^{C_n(x,t)} \qquad (1.5)$$

又级数（1.5）于D内一致收敛，关于t一次逐项求导．关于x二次逐项求导后的级数于D内仍一致收敛．

为简便，记（1.5）为

$$u(x,t)=\sum_{n=1}^{\infty}u_n(x,t) \qquad (1.5')$$

对（1.5′）逐项求导，且将$\dfrac{\partial u_n}{\partial t}$，$\dfrac{\partial u_n}{\partial x}$，$\dfrac{\partial^2 u_n}{\partial x^2}$以及$u(x,t)$的表达式代入（1.1）中，由于等式两端$\sin\dfrac{n\pi x}{l(t)}$，$\cos\dfrac{n\pi x}{l(t)}$的系数必然相等，所以有

$$B_n\frac{\partial C_n}{\partial t}-A_n(n\pi x)\frac{l'(t)}{l^2(t)}$$
$$=[-a^2(\frac{n\pi}{l(t)})^2+a^2\frac{\partial^2 C_n}{\partial x^2}+a^2(\frac{\partial C_n}{\partial x})^2+\frac{\gamma x+\delta}{a+\beta t}\frac{\partial C_n}{\partial t}-\frac{ex^2+hx+f}{(a+\beta t)^2}]$$

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$$+A_n[2a^2\frac{n\pi}{l(t)}\frac{\partial C_n}{\partial x}+\frac{\gamma x+\delta}{\alpha+\beta t}\cdot\frac{n\pi}{l(t)}], \tag{1.6}$$

$$A_n\frac{\partial C_n}{\partial t}+B_n(n\pi x)\frac{l'(t)}{l^2(t)}$$
$$=A_n[-a^2(\frac{n\pi}{l(t)})^2+a^2\frac{\partial^2 C_n}{\partial x^2}+a^2(\frac{\partial C_n}{\partial x})^2]+\frac{\gamma x+\delta}{\alpha+\beta t}\cdot\frac{\partial C_n}{\partial x}-\frac{ex^2+hx+f}{(\alpha+\beta t)^2}]$$
$$+B_n[-2a^2\frac{n\pi}{l(t)}\frac{\partial C_n}{\partial x}-\frac{\gamma x+\delta}{\alpha+\beta t}\frac{n\pi}{l(t)}]. \tag{1.7}$$

由(1.6)得，

$$\frac{\partial C_n}{\partial x}=a^2[-(\frac{n\pi}{l(t)})^2+\frac{\partial^2 C_n}{\partial x^2}+(\frac{\partial C_n}{\partial x})^2]+\frac{\gamma x+\delta}{\alpha+\beta t}\cdot\frac{\partial C_n}{\partial x}-\frac{ex^2+hx+f}{(\alpha+\beta t)^2}, \tag{1.8}$$

$$n\pi x\frac{l'(t)}{l^2(t)}=-2a^2\frac{n\pi}{l(t)}\frac{\partial C_n}{\partial x}-\frac{\gamma x+\delta}{\alpha+\beta t}\frac{n\pi}{l(t)}. \tag{1.9}$$

由(1.9)得

$$\frac{\partial C_n}{\partial x}=-\frac{x}{2a^2}\frac{l'(t)}{l(t)}-\frac{\gamma x+\delta}{2a^2(\alpha+\beta t)}. \tag{1.10}$$

对(1.10)关于x求导，得

$$\frac{\partial^2 C_n}{\partial x^2}=-\frac{1}{2a^2}\cdot\frac{l'(t)}{l(t)}-\frac{\gamma}{2a^2(\alpha+\beta t)}. \tag{1.11}$$

对(1.10)两端关于x积分，得

$$C_n(x,t)=-\frac{x^2}{4a^2}\frac{l'(t)}{l(t)}-\frac{\gamma x^2}{4a^2(\alpha+\beta t)}-\frac{\delta x}{2a^2(\alpha+\beta t)}+C_{n,0}(t). \tag{1.12}$$

将(1.10)，(1.11)代入(1.8)中，并积分之，得

$$C_n(x,t)=-a^2n^2\pi^2\int\frac{dt}{l^2(t)}-\frac{1}{2}\ln l(t)-\frac{\gamma}{2a^2}\int\frac{dt}{\alpha+\beta t}+\frac{x^2}{4a^2}\int\frac{l'^2(t)}{l^2(t)}dt$$
$$+\frac{(\gamma x+\delta)^2}{4a^2}\int\frac{dt}{(\alpha+\beta t)^2}+\frac{\gamma x^2+\delta x}{2a^2}\int\frac{l'(t)}{(\alpha+\beta t)^2 l(t)}dt$$
$$-\frac{\gamma x^2+\delta x}{2a^2}\int\frac{l'(t)}{(\alpha+\beta t)l(t)}dt-\frac{\gamma^2x^2}{2a^2}\int\frac{dt}{(\alpha+\beta t)^2}-\frac{\delta^2}{2a^2}\int\frac{dt}{(\alpha+\beta t)^2}$$
$$-\frac{\gamma\delta x}{a^2}\int\frac{dt}{(\alpha+\beta t)^2}-ex^2\int\frac{dt}{(\alpha+\beta t)^2}-hx\int\frac{dt}{(\alpha+\beta t)^2}$$
$$-f\int\frac{dt}{(\alpha+\beta t)^2}+C_{n,1}(x). \tag{1.13}$$

进而得

$$C_n(x,t)=-a^2n^2\pi^2\int\frac{dt}{l^2(t)}-\frac{1}{2}\ln l(t)+\frac{x^2}{4a^2}\int\frac{l'^2(t)}{l^2(t)}dt$$
$$-\frac{\gamma^2x^2}{4a^2}\int\frac{dt}{(\alpha+\beta t)^2}-\frac{\gamma}{2}\int\frac{dt}{\alpha+\beta t}-ex^2\int\frac{dt}{(\alpha+\beta t)^2}$$
$$-\frac{\gamma\delta x}{2a^2}\int\frac{dt}{(\alpha+\beta t)^2}-hx\int\frac{dt}{(\alpha+\beta t)^2}-\frac{\delta^2}{4a^2}\int\frac{dt}{(\alpha+\beta t)^2}$$
$$-f\int\frac{dt}{(\alpha+\beta t)^2}+C_{n,1}(x). \tag{1.13'}$$

由(1.12),(1.13′),有

$$C_{n,0}(t)=-a^2n^2\pi^2\int\frac{\mathrm{d}t}{l^2(t)}-\frac{1}{2}\ln l(t)-\frac{\gamma}{2}\int\frac{\mathrm{d}t}{a+\beta t}-\left(f+\frac{\delta^2}{4a^2}\right)\int\frac{\mathrm{d}t}{(a+\beta t)^2} \tag{1.14}$$

$$C_{n,1}(x)=0. \tag{1.15}$$

由(1.12),(1.13′)及条件 $h=-\frac{\delta\beta}{2a^2}\left(1+\frac{\gamma}{\beta}\right)$ 得

$$-\frac{x^2}{4a^2}\frac{l'(t)}{l(t)}-\frac{\gamma x^2}{4a^2(a+\beta t)}$$
$$=\frac{x^2}{4a^2}\int\frac{l'^2(t)}{l^2(t)}\mathrm{d}t-\frac{\gamma^2x^2}{4a^2}\int\frac{\mathrm{d}t}{(a+\beta t)^2}-ex^2\int\frac{\mathrm{d}t}{(a+\beta t)^2} \tag{1.16}$$

在(1.16)中约去公因子 x^2,并乘以 $4a^2$,且关于 t 求导得:

$$\frac{l''(t)l(t)-l'^2(t)}{l^2(t)}+\frac{a\gamma}{(a+\beta t)^2}=\frac{l'^2(t)}{l^2(t)}-\frac{\gamma^2}{(a+\beta t)^2}-\frac{4a^2e}{(a+\beta t)^2}$$ 亦即

$$(a+\beta t)^2l''(t)-(\gamma^2+a\gamma+4a^2e)l(t)=0. \tag{1.17}$$

设 $a+\beta t=e^v$,由此得

$$t=\frac{e^v-a}{\beta} \tag{1.18}$$

于是,

$$v=\ln(a+\beta t), \tag{1.19}$$

从而Euler方程(1.17)化为

$$\frac{\mathrm{d}^2l}{\mathrm{d}v^2}-\frac{\mathrm{d}\tilde{l}}{\mathrm{d}v}-\frac{\gamma^2+a\gamma+4a^2e}{\beta^2}\tilde{l}=0. \tag{1.20}$$

由特征方程

$$\lambda^2-\lambda-\frac{\gamma^2+a\gamma+4a^2e}{\beta^2}=0, \tag{1.21}$$

得特征根,

$$\lambda_{1,2}=\frac{1\pm\sqrt{1+4(\gamma^2+a\gamma+4a^2e)/\beta^2}}{2}.$$

所以有通解

$$l(t)=c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2} \tag{1.22}$$

由条件
$$\begin{cases} l(0)=c_1a^{\lambda_1}+c_2a^{\lambda_2}=l_0 \\ l(T)=c_1(a+\beta_T)^{\lambda_1}+c_2(a+\beta_T)^{\lambda_2}=l_T \end{cases}$$
得

$$c_1=\frac{\begin{vmatrix} l_0 & a^{\lambda_2} \\ l_T & (a+\beta_T)^{\lambda_2}\end{vmatrix}}{\begin{vmatrix} a^{\lambda_1} & a^{\lambda_2} \\ (a+\beta_T)^{\lambda_1} & (a+\beta_T)^{\lambda_2}\end{vmatrix}},\quad c_2=\frac{\begin{vmatrix} a^{\lambda_1} & l_0 \\ (a+\beta_T)^{\lambda_2} & l_T\end{vmatrix}}{\begin{vmatrix} a^{\lambda_1} & a^{\lambda_2} \\ (a+\beta_T)^{\lambda_1} & (a+\beta_T)^{\lambda_2}\end{vmatrix}}. \tag{1.23}$$

二、问题(A)有Fourier型解的充分必要条件

定理 问题(A)有Fourier型解的充分必要条件是可变长度 $l(t)=c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2}$

证明 1°）必要性：由上述分析可知，问题（A）有Fourier型解必然有$l(t)=c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2}$.

2°）充分性：设$T_0=[0,T]$，$E=\overline{D}\times T_0$，对$\forall(x_1,t_1),(x_2,t_2)\in E$，定义距离$\rho(x_1,t_1;x_2,t_2)=\sqrt{(x_1-x_2)^2+(t_1-t_2)^2}$．于是在$E$上构成距离空间，且为紧空间，仍记为$E$.设$C_R(E)$为$E$上的实连续函数集合．对$\forall\varphi_1,\varphi_2\in C_R(E)$，$c_1,c_2\in R$有$c_1\varphi_2+c_2\varphi_2\in C_R(E)$，对$\forall\varphi$，$\psi\in C_R(E)$，定义内积：$(\varphi,\psi)=\int_E\varphi\psi\mathrm{d}E$，范数：$\|\varphi\|^2=(\varphi,\varphi)=\int_E\varphi^2\mathrm{d}E$.

对$\forall\varphi_1,\varphi_2,\psi\in C_R(E)$，有

$$(\varphi_1,\varphi_2)=(\varphi_2,\varphi_1)\text{及}(c_1\varphi_1+c_2\varphi_2,\psi)=c_1(\varphi_1,\psi)+c_2(\varphi_2,\psi)$$

成立，$C_R(E)$ $L_2(E)$内积构成的空间不完备，经完备化而成为完备的Hilbert空间，记为$L_2(E)$空间.

设$M=\{u:u\in C^2(D)\cap C(\overline{D})\cap C(T_0)$，且$u(0,t)=u(l(t),t)=0\}$，对$\forall u_1,u_2\in M$，由于$c_1u_1+c_2u_2\in M$，$u_1,u_2\in M$，所以$M$为$L_2(E)$中的子代数．而对$\forall(x_1,t_1),(x_2,t_2)\in E$，若$t_1\neq t_2$，则$\exists\varphi\in M$，使$\varphi(x_1,t_1)\neq\varphi(x_2,t_2)$，从而隔离$E$的点，依Stone－Weierstrass定理，则M为$L_2(E)$中的稠密集合.

设 $Lu=\dfrac{\partial u}{\partial t}-a^2\dfrac{\partial^2u}{\partial x^2}-\dfrac{\gamma x+\delta}{a+\beta t}\dfrac{\partial u}{\partial x}+\dfrac{ex^2+hx+f}{(a+\beta t)^2}u$,对$\forall u\in M$，作内积

$$\begin{aligned}(Lu,u)=&\int_0^T\int_0^{c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2}}u\frac{\partial u}{\partial t}\mathrm{d}x\mathrm{d}t-a^2\int_0^T\int_0^{c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2}}u\frac{\partial^2u}{\partial x^2}\mathrm{d}x\mathrm{d}t\\&-\int_0^T\int_0^{c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2}}u\left(\frac{\gamma x+\delta}{a+\beta t}\frac{\partial u}{\partial x}\right)\mathrm{d}x\mathrm{d}t\\&+\int_0^T\int_0^{c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2}}u\left(\frac{ex^2+hx+f}{(a+\beta t)^2}u\right)\mathrm{d}x\mathrm{d}t\\\geq&\int_{c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2}-(c_1a^{\lambda_1}+c_2a^{\lambda_2})}^{c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2}}u^2(x,T)\mathrm{d}x\\&+\int_0^{c_1a^{\lambda_1}+c_2a^{\lambda_2}}[u^2(x,T)-u^2(x,0)]\mathrm{d}x+\frac{\gamma}{2}\int_0^T\int_0^{c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2}}u^2\mathrm{d}x\mathrm{d}t\\\geq&\frac{\gamma}{2}\|u\|^2_{L_2(E)}\quad.\end{aligned}\tag{2.1}$$

（2.1）表明算子L为定义于稠密集合M上的正定算子.

设$V\in M$，又$V(x,0)=F(x)$．令$W=u-v$．于是$W(0,t)=W(l(t),t)$，$W(x,0)=0$. 将$u=W+V$代入问题（A）中，得

$$(\mathrm{A}')\quad\begin{cases}\dfrac{\partial W}{\partial t}=a^2\dfrac{\partial^2W}{\partial x^2}+\dfrac{\gamma x+\delta}{a+\beta t}\dfrac{\partial W}{\partial x}-\dfrac{ex^2+hx+f}{(a+\beta t)^2}W+g(x,t)\\W(0,t)=W(l(t),t)=0,\\W(x,0)=0\end{cases}$$

其中

$$g(x,t)=a^2\frac{\partial^2V}{\partial x^2}+\frac{\gamma x+\delta}{a+\beta t}\frac{\partial V}{\partial x}-\frac{ex^2+hx+f}{(a+\beta t)^2}V-\frac{\partial V}{\partial t}.$$

由此，可将问题（A′）化为定义于稠密集合M上的算子方程$LW=g$. 由于算子L的正定性，则二次泛函$\Phi(W)=(LW,W)_{L_2(E)}-2(W,g)_{L_2(E)}$存在唯一的极小值. 依F.Riesz变分原理，问题（A′）有唯一解，亦即问题（A）有唯一的Fourier型解.

三、问题（A）Fourier型解

将（1.10），（1.11）代到（1.8）中，并从$t_0\to t$积分，其中$t_0, t\in(0,T)$).得

$$
\begin{aligned}
C_n(x,t)=&\frac{a^2n^2\pi^2}{c_2\beta}\left[\frac{1}{c_1+c_2(a+\beta t)^{-p}}-\frac{1}{c_1+c_2(a+\beta t_0)^{-p}}\right]\\
&-\ln\left[\frac{c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2}}{c_1(a+\beta t_0)^{\lambda_1}+c_2(a+\beta t_0)^{\lambda_2}}\right]^{\frac{1}{2}}\left(\frac{a+\beta t}{a+\beta t_0}\right)^{\frac{\gamma}{2\beta}}\\
&-\frac{1}{\beta}\left(\frac{\beta^2x^2-\gamma^2x^2}{4a^2}-\frac{\gamma\delta x}{2a^2}-hx-\frac{\delta^2}{4a^2}-ex^2-f\right)\left(\frac{1}{a+\beta t}-\frac{1}{a+\beta t_0}\right).
\end{aligned}
\tag{3.1}
$$

其中$p=\sqrt{1+4(\gamma^2+a\gamma+4a^2e)/\beta^2}$. 将（3.1）代到（1.5）中，得

$$
\begin{aligned}
u(x,t)=&\left[\frac{c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2}}{c_1(a+\beta t_0)^{\lambda_1}+c_2(a+\beta t_0)^{\lambda_2}}\right]^{-\frac{1}{2}}\left(\frac{a+\beta t}{a+\beta t_0}\right)^{-\frac{\gamma}{2\beta}}\\
&\cdot\exp\left\{\frac{1}{\beta}\left[\left(\frac{\beta^2-\gamma^2}{4a^2}-e\right)x^2-\left(\frac{\gamma\delta}{2a^2}+h\right)x-\left(\frac{\delta^2}{4a^2}+f\right)\right]\left(\frac{1}{a+\beta t_0}-\frac{1}{a+\beta t}\right)\right\}\\
&\cdot\sum_{n=1}^{\infty}\left[A_n\sin\frac{n\pi x}{c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2}}+B_n\cos\frac{n\pi x}{c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2}}\right]\\
&\cdot\exp\frac{a^2n^2\pi^2}{c_2\beta}\left[\frac{1}{c_1+c_2(a+\beta t)^{-p}}-\frac{1}{c_1+c_2(a+\beta t_0)^{-p}}\right].
\end{aligned}
\tag{3.2}
$$

（i） 依边界条件$u(0,t)=0$，得$B_n=0$.

（ii） 依初始条件$u(x,0)=F(x)$，得

$$
\begin{aligned}
A_n=&\frac{2}{l_0^2}\left[\frac{c_1a^{\lambda_1}+c_2a^{\lambda_2}}{c_1(a+\beta t_0)^{\lambda_1}+c_2(a+\beta t_0)^{\lambda_2}}\right]^{\frac{1}{2}}\left(\frac{a}{a+\beta t_0}\right)^{\frac{\gamma}{2\beta}}\\
&\cdot\exp\frac{a^2n^2\pi^2}{c_2\beta}\left[\frac{-1}{c_1+c_2a^{-p}}+\frac{1}{c_1+c_2(a+\beta t_0)^{-p}}\right]\\
&\cdot\int_0^{l_0}F(x)\exp\left\{\frac{1}{\beta}\left[\left(\frac{\beta^2-\gamma^2}{4a^2}-e\right)x^2-\left(\frac{\gamma\delta}{2a^2}+h\right)x-\left(\frac{\delta^2}{4a^2}+f\right)\right]\left(\frac{1}{a}-\frac{1}{a+\beta t_0}\right)\right\}\\
&\cdot\sin\frac{n\pi x}{l_0}\mathrm{d}x
\end{aligned}
$$

再讨论（3.2）. 它是否是问题（A）Fourier型的解呢？设

$$
\begin{aligned}
\tilde{u}(x,t)=&\sum_{n=1}^{\infty}A_n\sin\frac{n\pi x}{c_1(a+\beta t)^{\lambda_1}+c_2(a+\beta t)^{\lambda_2}}\\
&\cdot\exp\frac{a^2n^2\pi^2}{c_2\beta}\left[\frac{1}{c_1+c_2(a+\beta t)^{-p}}-\frac{1}{c_1+c_2(a+\beta t_0)^{-p}}\right],
\end{aligned}
\tag{3.3}
$$

从而

$$|\widetilde{u}(x,t)|\le\sum_{n=1}^{\infty}|A_n|\exp\frac{a^2n^2\pi^2}{c_2\beta}\left[\frac{1}{c_1+c_2(a+\beta t)^{-p}}-\frac{1}{c_1+c_2(a+\beta t_0)^{-p}}\right]$$

$$\le\frac{2}{l_0}\left[\frac{c_1a^{\lambda_1}+c_2a^{\lambda_2}}{c_1(a+\beta t_0)^{\lambda_1}+c_2(a+\beta t_0)^{\lambda_2}}\right]^{\frac{1}{2}}\left(\frac{a}{a+\beta t_0}\right)^{\frac{\gamma}{2\beta}}$$

$$\cdot\sum_{n=1}^{\infty}\exp\frac{a^2n^2\pi^2}{c_2\beta}\left[\frac{1}{c_1+c_2(a+\beta t)^{-p}}-\frac{1}{c_1+c_2(a+\beta t_0)^{-p}}\right]$$

$$\cdot\int_0^{l_0}|F(x)|\exp\left\{\frac{1}{\beta}\left[\left(\frac{\beta^2-\gamma^2}{4a^2}-e\right)x^2-\left(\frac{\gamma\delta}{2a^2}+h\right)x-\left(\frac{\delta^2}{4a^2}+f\right)\right]\right.$$

$$\left.\cdot\left(\frac{1}{a}-\frac{1}{a+\beta t_0}\right)\right\}\left|\sin\frac{n\pi x}{l_0}\right|dx.$$

1°) $\int_0^{l_0}|F(x)|\exp\varphi(x)\left|\sin\frac{n\pi x}{l_0}\right|dx\le M_1\int_0^{l_0}\exp\varphi(x)\frac{n\pi x}{l_0}dx=\frac{n\pi M_1}{l_0}h_1$

其中

$$|F(x)|\le M_1,\quad h_1=\int_0^{l_0}[\exp\varphi(x)]x\,dx,$$

$$\varphi(x)=\frac{1}{\beta}\left[\left(\frac{\beta^2-\gamma^2}{4a^2}-e\right)x^2-\left(\frac{\gamma\delta}{2a^2}+h\right)x-\left(\frac{\delta^2}{4a^2}+f\right)\right]\left(\frac{1}{a}-\frac{1}{a+\beta t_0}\right)].$$

2°) 设 $t_0<t_1<t$，则

$$\exp\frac{a^2n^2\pi^2}{c_2\beta}\left[\frac{1}{c_1+c_2(a+\beta t)^{-p}}-\frac{1}{c_1+c_2(a+\beta t_0)^{-p}}\right]$$

$$<\exp\frac{a^2n^2\pi^2}{c_2\beta}\left[\frac{1}{c_1+c_2(a+\beta t_1)^{-p}}-\frac{1}{c_1+c_2(a+\beta t_0)^{-p}}\right]=\exp(-\mu^2n^2).$$

其中 $\mu^2=\frac{a^2\pi^2}{\beta}\frac{(a+\beta t_1)^{-p}-(a+\beta t_0)^{-p}}{[c_1+c_2(a+\beta t_1)^{-p}][c_1+c_2(a+\beta t)^{-p}]}$．于是

$$|\widetilde{u}(x,t)|<\frac{2}{l_0^2}\left(\frac{c_1a^{\lambda_1}+c_2a^{\lambda_2}}{c_1(a+\beta t_0)^{\lambda_1}+c_2(a+\beta t_0)^{\lambda_2}}\right)^{\frac{1}{2}}\left(\frac{a}{a+\beta t_0}\right)^{\frac{\gamma}{2\beta}}\pi M_1h\cdot\sum_{n=1}^{\infty}ne^{-\mu^2n^2}$$

而级数 $\sum_{n=1}^{\infty}ne^{-\mu^2n^2}$ 收敛，所以(3.3)于 D 内一致收敛．同理(3.2)于 D 内一致收敛．同样可以证明(3.2)关于 t 逐项一次求导以及关于 x 一、二次逐项求导后的级数于 D 内一致收敛．还可以证明(3.2)满足问题(A)，于是Fouriev型级数(3.2)是问题(A)的解．

参 考 文 献

[1] 周本濂，吉新华，应用数学学报，Vol.10，No.1，(1987)，101—105.

[2] Dong, L., Bai, L., Zhou, B. L., A Method to Obtain the Analytic Solution of the Moving Baundary Problem, the Symposim of the 17th International Thermal Canductivity Conberence, 1981.

[3] J. Diendonne, Foundations of modern analysis, 136—139, New Youk and London, 1969.

The Moving Boundary Problem of Heat-Conduction Equation

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Summary

The heat-conduction problem is often a movable boundary problem under the high temperature. A minute description of the method used to solve the moving boundary problem which is attached to the phsyical process such as gradual burning of a wire is give in [1]. This paper gives a necessary and sufficient condition under which the movable boundary problem of general heat-conduction eqation has Fourier form solution, and presents the solution of the problem.