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The Binding Number of a Graph*

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The binding number of a graph is an important characteristic quantity of a graph. In 1973 Woodall first introduced the concept of the binding number of a graph and then gave the binding number of some special graphs in [1]. In 1981 Kane, Mohanty and Hales gave the binding number of some product graphs in [2]. Wang Jianfang, Tian Songlin, Liu Ziuqiang[3,4] (1983—1987) gave the binding number of some multi-product graphs, and it is the limit character. Up to now, We have given the binding number of the following types of graphs:

$K_n, K_{a,b}, C_n, L_n$; $K_m\times K_n$, $L_m\times L_n, C_m\times C_n$, $K_{a,b}\times L_n$, $K_{a,b}\times C_n$, $K_n\times K_{a,b}, L_n\times C_m$, $L_{m_1}\times L_{m_2}\times\cdots\times L_{m_n}$, $K_{m_1}\times C_{m_2}\times\cdots\times C_{m_n}$, $K_{m_1}\times L_{m_2}\times\cdots\times L_{m_n}$, $C_{m_1}\times C_{m_2}\times\cdots\times C_{m_n}$, $K_{a,b}\times C_{m_1}\times\cdots\times C_{m_n}$; $K_m\otimes K_n$, $L_m\otimes K_n, K_{a,b}\otimes L_n$, $K_{a,b}\otimes C_n$, $K_{a,b}\otimes K_n$, $K_{a,b}\otimes K_{m,n}, C_m\otimes K_n$, $C_m\otimes C_n$, $C_{m_1}\otimes C_{m_2}\otimes\cdots\otimes C_{m_n}$, $L_{m_1}\otimes L_{m_2}\otimes\cdots\otimes L_{m_n}$; $K_m*K_n, K_{a,b}*K_n$, L_m*K_n, C_m*K_n, $L_{m_1}*L_{m_2}*\cdots*L_{m_n}$, $C_{m_1}*C_{m_2}*\cdots*C_{m_n}$, $L_{m_1}*\cdots*L_{m_r}*K_n$, $C_{m_1}*\cdots*C_{m_r}*K_n$; $K_m(K_n)$, $K_{a,b}(K_n)$, $L_m(K_n)$, $C_m(K_n)$, $K_n(K_{a,b})$, $K_m(L_n)$, $K_m(C_n)$.

Generally, how to calculate the binding number of a graph? This problem has not be solved so far. In the following we have given some new results. Other terms and notations not be defined in this paper can be found in[1, 2, 3].

The Main Results

Let $F=\{X|\phi\neq X\subseteq V(G), \Gamma(X)\neq V(G)\}$, $G=L_{m_1}\times C_{m_2}\times\cdots\times C_{m_n}$.

Lemma 1 For any $X\in F$, $|X|\leq\prod_{i=1}^{n}m_i-2n+1$.

Lemma 2 If exist $i\in\{2,3,\cdots,n\}$ such that m_i is odd. The n

$$|\Gamma(X)|\geq|X|+2n-2, \text{ for any } X\in F.$$

By lemma 1, lemma 2 and Corollary 7.1 in [1]. We can obtaine the following theorem.

Theorem 3 Let $G=L_{m_1}\times C_{m_2}\times\cdots\times C_{m_n}$. Then

$$\operatorname{bind}(G)=\begin{cases}1, & \text{if all } m_i\ (2\leq i\leq n) \text{ are even},\\ \left(\prod_{i=1}^{n}m_i-1\right)/\left(\prod_{i=1}^{n}m_i-2n+1\right), & \text{otherwise}.\end{cases}$$

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Corollary 4 Let $G = L_{n_r} \times L_{n_{r-1}} \times \cdots \times L_{n_1} \times C_{m_1} \times \cdots \times C_{m_s}$. Then

$$\mathrm{bind}(G) = \begin{cases} 1, & \text{if all } m_j (1 \leq j \leq s) \text{ are even}, \\ \left(\left(\prod_{i=1}^{r} n_i\right)\left(\prod_{j=1}^{s} m_j\right) - 1\right) / \left(\left(\prod_{i=1}^{r} n_i\right)\left(\prod_{j=1}^{s} m_j\right) - (2s+r)\right), & \text{otherwise}. \end{cases}$$

Corollary 5 Let $G_n = G_{p_1} \times G_{p_2} \times \cdots \times G_{p_n}$, where p_i $(1 \leq i \leq n)$ are positive integers. If $G_{p_i} = L_{p_i}$ $(p_i \geq 2)$; or $G_{p_i} = C_{p_i}$ $(p_i \geq 3)$; or $G_{p_1} = K_{p_1}$ and $G_{p_i} = C_{p_i}$ $(p_i \geq 3, 2 \leq i \leq n)$; or $G_{p_1} = K_{p_1}$ and $G_{p_i} = L_{p_i}$ $(p_i \geq 2, 2 \leq i \leq n)$; or $G_{p_1} = K_{a,b}$ and $G_{p_i} = C_{p_i}$ $(p_i \geq 3, 2 \leq i \leq n)$; or $G_{p_1} = L_{p_1}$ and $G_{p_i} = C_{p_i}$ $(p_i \geq 3, 2 \leq i \leq n)$. Then $\lim_{n \to \infty} \mathrm{bind}(G_n) = 1$.

Corollary 6 Let $G_n = G_{p_1} \otimes C_{p_2} \otimes \cdots \otimes C_{p_n}$, $G_m = L_{q_1} * L_{q_2} * \cdots * L_{q_m}$, $G_k = C_{h_1} * C_{h_2} * \cdots * C_{h_k}$. where $p_i \geq 3$ $(1 \leq i \leq n)$, $q_j \geq 3$ $(1 \leq j \leq m)$ and $h_r \geq 4$ $(1 \leq r \leq k)$ are integers. Then

$$\lim_{n \to \infty} \mathrm{bind}(G_n) = \lim_{m \to \infty} \mathrm{bind}(G_m) = \lim_{k \to \infty} \mathrm{bind}(G_k) = 1.$$

Theorem 7 Let n_1, n_2, n_r $(r \geq 2)$ are positive integers and $n_1 \leq n_2 \leq \cdots \leq n_r$. Then

$$\mathrm{bind}(K_{n_1, n_2, \cdots, n_r}) = (n_1 + n_2 + \cdots + n_{r-1}) / n_r.$$

Theorem 8 Let a, b, n and m are positive integers. Then $\mathrm{bind}(K_{a,b}(\overline{K}_n)) = \min\{a/b, b/a\}$.

$$\mathrm{bind}(C_m(\overline{K}_n)) = \begin{cases} 0, & \text{if } m = 1. \\ 1, & \text{if } m > 1 \text{ and } m \text{ is even}, \\ (m-1)/(m-2), & \text{if } m > 1 \text{ and } m \text{ is odd}. \end{cases}$$

$$\mathrm{bind}(L_m(\overline{K}_n)) = \begin{cases} 0, & \text{if } m = 1, \\ 1, & \text{if } m > 1 \text{ and } m \text{ is even}, \\ (m-1)/(m+1), & \text{if } m > 1 \text{ and } m \text{ is odd}. \end{cases}$$

Finally, we propose the following conjecture.

Conjecture Let a, b, m, n are positive integers and $b > a > 1$, $n > m > 1$. Then

$$\mathrm{bind}(K_{a,b} \times K_{m,n}) = ((a+b)(m+n) - 1)/((a+b)(m+n) - (a+m)).$$

References

[1] D. R. Woodall, The Binding Number of a Graph and its Anderson Number, J. Combinatorial Thegry, Ser. B 15(1973), 225—255.

[2] V. G. Kane, S. P. Mohanty and R. S. Hales, ARS Combinatoria, 11(1981), 201—204.

[3] Wang Jianfang, Tian Songlin, Liu Ziuqiang, The Binding Number of Product graphs, Lecture Notes in Mathematics 1073 Graph Theory Singapore 1983 Proceding 119→128.

[4] Wang Jianfang, Tian Songlin, Liu Ziuqiang, The problem about the Binding Number of a Graph, Acta Mathematica Sinica, Vol. 30, No.5 (1987), 653—660.