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Three-dimensional Pythagorean Theorem for Elliptic Space

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Let M be an elliptic space with constant curvature $K = 1/R^2$, and consider in M a right-angled tetrahedron, i.e., a tetrahedron with three mutually perpendicular edges at a certain vertex. Denote the tetrahedron by VABC and assume that each of the edges VA, VB and VC is perpendicular to others. The areas of triangles VBC, VCA, VAB and ABC are denoted by S_1, S_2, S_3 and S, respectively. We obtain the following

Theorem $$\cos\frac{S}{2R^2} = \cos\frac{S_1}{2R^2}\cos\frac{S_2}{2R^2}\cos\frac{S_3}{2R^2} + \sin\frac{S_1}{2R^2}\sin\frac{S_2}{2R^2}\sin\frac{S_3}{2R^2}.$$

This theorem is an analogue of the well known Pythagorean theorem for Euclidean plane.

When the elliptic space has a large curvature radius R, the theorem becomes approximately

$$S^2 = S_1^2 + S_2^2 + S_3^2,$$

which is the three-dimensional Pythagorean theorem for Euclidean space.