Journal of Mathematical Research & Exposition,
Vol.12, No.3, Aug. 1992

Global Pinching Theorems of Submanifolds with Constant Mean Curvature in the Sphere*

Sun Huafei
(Dept. of Math., Northeast University of Technology, Shenyang, China)

Cui Yuheng
(Dept. of Math., Liaoning University, Shenyang, China)

Let $S^{n+p}(1)$ be a $(n+p)$-dimensional unit sphere. We prove the following results.

Theorem 1 *Let $M^n(n>2)$ be a compact oriented n-dimensional submanifold with parallel mean curvature vector in $S^{n+p}(1)(p>1)$, Ricci curvature of $M^n>(n-1)c, 0<c\leq 1$(c is a constant). Then there is a constant $A(n,c,H)$, such that if*

$$\|\sigma\|_{\frac{n}{2}}<A(n,c,H),$$

then M^n is a totally umbilical submanifold. Where

$$A(n,c,H) = \min\left\{[\frac{n-2}{2(n-1)}]^2\frac{2^{-\frac{n+6}{n}}}{1+\sqrt{n-1}}[\frac{[\alpha(n)]^2}{\alpha(n-1)}\frac{n^{n+1}c^{n(n+1)/2}}{[\pi^2(1+H^2)]^{n(n+1)/2}}]^{\frac{2}{n}},\right.$$
$$\left.\frac{n}{1+\sqrt{n-1}}[\alpha(n)]^{\frac{2}{n}}2^{-[\frac{n+4}{n}+\varepsilon(n)]}\right\}.$$

Theorem 2 *Let M^2 be a compact oriented 2-dimensional submanifold with parallel mean curvature vector in $S^{2+p}(p>2)$, Gauss curvature of $M^2>c, 0<c\leq 1$(c is a constant). If*

$$\|\sigma\|_2<\frac{2c^{7/2}}{\pi^{11/2}(1+H^2)^3},$$

then M^2 is a totally umbilical submanifold.

Where σ and H denote the square of the length of the second fundamental form of M^n and the mean curvature of M^n respectively. $\alpha(n)$ is the volume of n-dimensional unit sphere in $R^{n+1}\cdot\|\sigma\|_k=(\int\sigma^k dv)^{\frac{1}{k}}, \varepsilon(n)=\begin{cases}1 & n=3\\ (n-4)(n-2)/2 & n>3.\end{cases}$

*Received Nov. 16, 1990.