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Sum Property, Normal Structure and LD Property of Orlicz Sequence Spaces*

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It is well known that (weakly) normal structure, (weak) sum-property, LD property and G_ε property are the fundamental tools in fixed points theory of nonexpansive mappings. Let X be a Banach space, $(x_n)_{n\in\mathrm{N}}$ be a bounded sequence of X. If for any point X belonging to the convex hull $\mathrm{covx}((x_n)_{n\in\mathrm{N}})$ of $(x_n)_{n\in\mathrm{N}}$, there holds

$$\overline{\lim_n}\sup \| x_n - x \| = \underline{\lim_n}\inf \| x_n - x \| \triangleq \wedge(x),$$

where $\wedge(\cdot)$ is an affine functional defined on $\mathrm{covx}((x_n)_{n\in\mathrm{N}})$; we call $(x_n)_{n\in\mathrm{N}}$ to be a limit-affine sequence. In particular, if $\wedge(\cdot)$ is a constant number on $\mathrm{covx}((x_n)_{n\in\mathrm{N}})$, we call $(x_n)_{n\in\mathrm{N}}$ to be a limit-constant sequence. Moreover, X is said to have (weak) sum-property if X contains no (weakly convergent) limit-affine sequence which $(\wedge x_n)_{n\in\mathrm{N}}$ is nondecreasing. X is said to have (weakly) normal structure if it contains no (weakly convergent) limit-constant sequence. X is said to have G_ε property if for any $\varepsilon, 0<\varepsilon<1$, there exists $r>0$ such that $\| x \|=1, \| y \| \geq \varepsilon$ and x, y have the support sets disjoint to each other implies $\| x+y \| \geq 1+r$. X is said to have Lami-Dozo property if any bounded sequence $(x_n)_{n\in\mathrm{N}}$ of X has a subsequence which is pointwise and almost convergent. We call that $(x_n)_{n\in\mathrm{N}}$ is almost convergent to x if for any $y \neq x$, $\overline{\lim}_n \sup_n \| x_n - x \| < \underline{\lim}_n \inf_n \| x_n - y \|$. The relations among these concepts are as shown in the following figure:

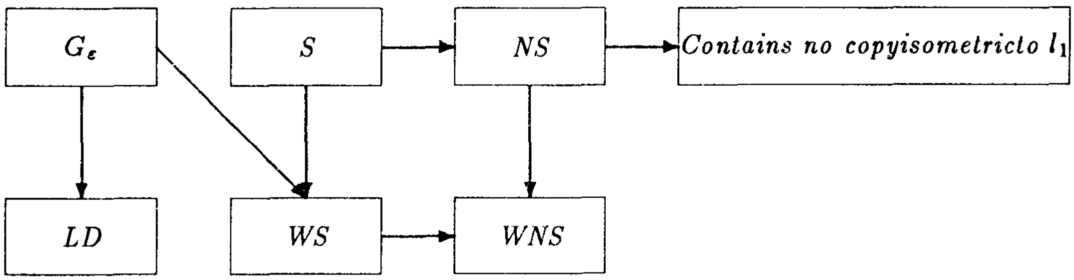

Fig. 1

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On Orlicz space $l_{(M)}$, endowed with Luxemburg norm, if $M(u)$ is restricted to be $N-$function, then every property listed above is equivalent to that $M(u)$ satisfies Δ_2-condition at 0. We discuss Orilicz sequence space l_M endowed with Orlicz norm. Both the results and the proof methods are quite different. Let $M(u), N(v)$ be a pair of complementary $N-$functions. We call that $[u, \bar{u}]$ is an affine interval of $M(u)$ if $M'(\tilde{u})$ equals to a constant for $\tilde{u} \in [u, \bar{u}]$.

Theorem 1 *The following are equivalent*

(1) l_M *has sum-property,*

(2) l_M *has normal structure,*

(3) *There exists* $c > 0$ *such that for any affine interval* $[u, \bar{u}]$ *of* $M(u)$ *containing in* $[0, N'(N^{-1}(1))]$, $\bar{u} \le cu$.

Theorem 2 l_M *has weak sum-property.*

Corollary 1 l_M *has weakly normal structure.*

Corollary 2 l_M *has the fixed point property.*

Theorem 3 *The following are equivalent*

(1) l_M *has* G_ε *property,*

(2) l_M *has LD property,*

(3) $M(u)$ *satisfies* Δ_2-*condition at* 0.

Remark l_M contains no subspace isometric to l_1.