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An Answer to the Question by K.L. Chung*

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Let $P = (p_{ij})$ be the transition probability matrix of a Markov chain. A probability distribution v^0 is called to have a history of length of n w.r.t P, where $n \in N$, if there exists a probability distribution v^{-n} such that $v^{-n}P^n = v^0$. If for any $n \in N, v^0$ has a history of length of n, then we say that v^0 has a history of infinite length w. r. t the transition probability matrix P.

Can a probability distribution have a history of infinite length? This question was raised by L.K. Hua. The following result was proved by Chung[1]:

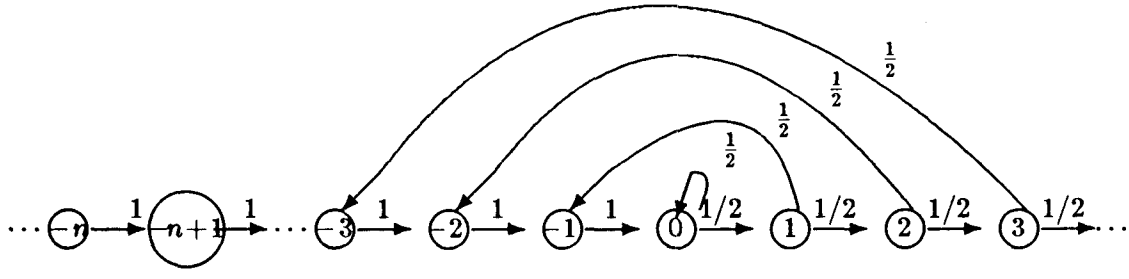
For a finite MC with transition probability matrix $P = (p_{ij})$,the probability distribution v^0 has a history of infinite length if and only if v^0 is periodic, namely, $\exists d \in N$, such that $v^0P^d = v^0$. Chung asked if the above result is still true for an infinite MC?

Unfortunately, we have the following counterexample:

Let the state space be the set of all integers, and the transition probability be as follows:

$$P_{ij} = \begin{cases} 1 & i < 0, \text{ and } j = i+1 \\ 1/2 & i \geq 0, \text{ and } j = -i \\ 1 & i \geq 0, \text{ and } j = i+1 \end{cases}.$$

The following graph shows the transition:



It is quite obvious that any two states communicate. Let $v = \{v_m, m \in Z\}$, where $v_m = 1/3 \times 2^{[m]}$, we can verify that $vP = v$. Hence the state space forms a positive recurrent class by Theorem in [2]. Now let $v^n = \{\delta_m^n, m \in Z\}$ where

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$$\delta_m^n = \begin{cases} 1 & \text{if } n = m \\ 0 & \text{if } n \neq m \end{cases}.$$

Then it is easy to see that $v^{-n}P = v^0$ for all $n \geq 1$. Thus v^0 has infinite history but it is not periodic.

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References

[1] Chung Kai-lai, *Markov chain must have a beginning*, J. Math. Res. & Exposition, 1986, No 1.

[2] Chung Kai-lai, *Lectures on Markov chains and stochastic processes* (Dalian, 1991).

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要摘

钟先生指出，对有限的MC，概率分布v^0有无穷长历史当且仅当v^0是有周期的，本文构造了一个反例，证明该结论不能简单地推广到无穷MC.