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Submanifolds with Parallel Mean Curvature Vector*

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In a previous paper [1], S.T. Yau proved the following theorem:

Theorem A *Let M^n be an n-dimensional compact submanifold with parallel mean curvature in S^{n+p} with $p > 1$. If $(3+\sqrt{n}-\frac{1}{p-1})S \leq n$, then M^n lies in a totally geodesic S^{n+1}.*

Lemma 1[2] *If a given set of $n+1$ $(n \geq 2)$ real numbers $a_1, \cdots, a_n$ and k satisfy the inequality*

$$\sum_{i=1}^{n} a_i^2 + k^2 < \frac{1}{n-1}(\sum_{i=1}^{n} a_i)^2.$$

Then for any pair of distinct i and $j = 1, \cdots, n$, we have

$$k^2 < 2a_i a_j.$$

Lemma 2[2] *For any set of $2n$ real numbers $\{a_1, \cdots, a_n, b_1, \cdots, b_n\}$ we have*

$$(\sum_{i=1}^{n} a_i)(\sum_{j=1}^{n} a_j b_j^2) - (\sum_{i=1}^{n} a_i b_i)^2 = \sum_{i<j}^{n} a_i a_j (b_i - b_j)^2.$$

By virtue of Lemma 1 and Lemma 2, we improve the pinching constant of Theorem A as the following result:

Theorem *Let M^n be an n-dimensional oriented closed submaonifold with parallel mean curvature in S^{n+p} with $p > 1$. If $(2-\frac{1}{p-1})/(\frac{n}{n-1}+(1-\frac{1}{p-1}))s < n$, then M^n lies in a totally geodesic S^{n+1}.*

References

[1] S.T. Yau, *Submanifolds with constant mean curvature, II*, Amer. J. Math., **97**(1975), 76–100.

[2] M. Okumura, *Submanifolds and a pinching problem on the second fundamental tensors*, Trans. Amer. Math. Soc., **178**(1973), 285– 291.

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