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一类修正的KdV方程的特征问题*

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摘　要

本文将一类修正的KdV方程

$$u_t+\alpha uu_x+\varepsilon u^2u_x+\mu u_{x^3}+\beta(1+u^2)(1+u_x^2)(1+u_t^2)(1+u_{x^3}^2)(u_{x^3}+\sigma u_{x^3t})=0$$

的特征问题，化为与之等价的积分微分方程，并依不动点原理，由该积分微分方程序列而得到此类修正的KdV方程特征问题在 $\bar{\Omega}$ 上一致收敛的迭代解.

一、设计结构

在一类修正的KdV方程的特征问题

$$(\mathrm{T})\quad\begin{cases}u_t+\alpha uu_x+\varepsilon u^2u_x+\mu u_{x^3}+\beta(1+u^2)(1+u_x^2)(1+u_t^2)(1+u_{x^3}^2)(u_{x^3}+\sigma u_{x^3t})=0\\ u(x,0)=h(x)\\ u(a,t)=f(t)\\ u_x(a,t)=g(t)\\ u_{x^2}(a,t)=\varphi(t)\end{cases}$$

中，$\bar{\Omega}=\{(x,t):a\leqslant x\leqslant b,\ 0\leqslant t\leqslant T_0,\ a,b,T_0\in R^+\}$，$h(x)\in C^3[a,b]$，$f(t),g(t),\varphi(t)\in C^1[0,T_0]$，又 $f(0)=h(a)$，$g(0)=h'(a)$，$\varphi(0)=h''(a)$，$\frac{5}{8|\beta|}(1+\frac{|\alpha|}{2}+|\varepsilon|+|\mu|)<\frac{\sigma-1}{2}<\frac{1}{2}$，$1<\sigma<2$.

对方程变形，且利用算子形式表示

$$\begin{aligned}\mathscr{L}(u)&=-\frac{1}{2}(u_{x^3}+u_{x^3t})\\&=\frac{1}{2\beta(1+u^2)(1+u_x^2)(1+u_t^2)(1+u_{x^3}^2)}(u_t+\alpha uu_x+\varepsilon u^2u_x+\mu u_{x^3})+\frac{\sigma-1}{2}u_{x^3t}\\&=F(u,u_x,u_t,u_{x^3},u_{x^3t}).\end{aligned}\tag{1.1}$$

于是特征问题化为

$$(\mathrm{T}')\quad\begin{cases}\mathscr{L}(u)=F(u,u_x,u_t,u_{x^3},u_{x^3t})\\ u(x,0)=h(x)\\ u(a,t)=f(t)\\ u_x(a,t)=g(t)\\ u_{x^2}(a,t)=\varphi(t)\end{cases}$$

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算子 $\mathscr{L}(u)$ 的共轭算子为

$$\mathscr{L}^*(v) = \frac{1}{2}v_{x^3} - \frac{1}{2}v_{x^3t}. \tag{1.2}$$

对 $\forall\ (x_0,t_0)\in\bar{\Omega}$,问题

$$(\mathrm{R})\qquad \begin{cases} \mathscr{L}^*(v) = 0 \\ v(x_0,t;x_0,t_0) = 0 \\ v(x,t_0;x_0,t_0) = (x-x_0)^2 \\ v_x(x_0,t;x_0,t_0) = 0 \\ v_x(x,t_0;x_0,t_0) = 2(x-x_0) \\ v_{x^2}(x_0,t;x_0,t_0) = 2e^{2(t-t_0)} \\ v_{x^2}(x,t_0;x_0,t_0) = 2 \end{cases}$$

的解 $v(x,t;x_0,t_0)=(x-x_0)^2e^{2(t-t_0)}$ 为方程 $\mathscr{L}(u)=0$ 的 Riemann 函数.

设

$$C_0^4(\bar{\Omega}) = \{u : u, u_x, u_t, u_{x^3}, u_{x^3t}\in C(\bar{\Omega})\} \tag{1.3}$$

并于 $C_0^4(\bar{\Omega})$ 上定义范数

$$\|u\|_\lambda = \max\{(|u| + |u_x| + |u_t| + |u_{x^3}| + |u_{x^3t}|)\exp[-\lambda(x+t)]\}, \tag{1.4}$$

其中 λ 为充分大的正常数,可知 $C_0^4(\bar{\Omega})$ 为 Banach 空间.

对任意给定的 $\bar{u}\in C_0^4(\bar{\Omega})$,讨论辅助问题,

$$(\mathrm{T}'')\qquad \begin{cases} \mathscr{L}(u) = F(\bar{u},\bar{u}_x,\bar{u}_t,\bar{u}_{x^3},\bar{u}_{x^3t}) \\ u(x,0) = h(x) \\ u(a,t) = f(t) \\ u_x(a,t) = g(t) \\ u_{x^2}(a,t) = \varphi(t) \end{cases}$$

设计结构

$$\begin{aligned} v\mathscr{L}(u) - u\mathscr{L}^*(v) &= \frac{\partial}{\partial x}[(vu_{x^2t} - v_xu_{xt} + v_{x^2}u_t) + 2(vu_{x^2} - v_xu_x + v_{x^2}u)] \\ &\quad - \frac{\partial}{\partial x}(\frac{3}{2}vu_{x^3}) + \frac{1}{2}(vu_{x^2}) - \frac{1}{2}v_xu_{x^2} \\ &= -\frac{1}{2}v(u_{x^3} + u_{x^3t}) = vF(\bar{u},\bar{u}_x,\bar{u}_{x^3},\bar{u}_{x^3t}). \end{aligned} \tag{1.5}$$

设 $\bar{D}=\{(x,t): a\leqslant x\leqslant x_0\leqslant b,\ 0\leqslant t\leqslant t_0\leqslant T_0\}$,边界为 Γ_0. 将(1.5)两端于 $\bar{D}$ 上积分,并利用 Green 公式及定解条件,得

$$\begin{aligned} &\int_0^{t_0}\int_a^{x_0}(x-x_0)^2e^{2(t-t_0)}F(\bar{u},\bar{u}_x,\bar{u}_t,\bar{u}_{x^3},\bar{u}_{x^3t})dxdt \\ &= \int_0^{t_0}\int_a^{x_0}\{\frac{\partial}{\partial x}[vu_{x^2t} - v_xu_{xt} + v_{x^2}u_t) + 2(vu_{x^2} - v_xu_x + v_{x^2}u)] - \frac{\partial}{\partial t}(\frac{3}{2}vu_{x^3})\}dxdt \\ &\quad + \frac{1}{2}\int_0^{t_0}\int_a^{x_0}\frac{\partial}{\partial x}(vu_{x^2})dxdt - \frac{1}{2}\int_0^{t_0}\int_a^{x_0}v_xu_{x^2}dxdt \\ &= \oint_{\Gamma_0}(\frac{3}{2}vu_{x^3})dx + [(vu_{x^2t} - v_xu_{xt} + v_{x^2}u_t) + 2(vu_{x^2} - v_xu_x + v_{x^2}u)]dt \end{aligned}$$

$$+\frac{1}{2}\int_0^{t_0}\int_a^{x_0}\frac{\partial}{\partial x}(vu_{z^2})dxdt-\frac{1}{2}\int_0^{t_0}\int_a^{x_0}v_x u_{z^2}dxdt$$

$$=e^{-2t_0}[h(x_0)-h(a)]+(a-x_0)e^{-2t_0}-\frac{1}{2}(a-x_0)^2e^{-2t_0}h''(a)-(a-x_0)g(t_0)$$

$$+\frac{1}{2}(a-x_0)^2\varphi(t_0)-u(x_0,t_0)+f(t_0)+(a-x_0)\int_0^{t_0}e^{2(t-t_0)}g(t)dt$$

$$-\frac{1}{2}(a-x_0)^2\int_0^{t_0}e^{2(t-t_0)}\varphi(t)dt+\int_0^{t_0}e^{2(t-t_0)}[u(x_0,t)-f(t)]dt. \tag{1.6}$$

(1.6)两端对 t_0 求导数，得

$$\int_a^{x_0}(x-x_0)^2F(\bar{u},\bar{u}_x,\bar{u}_t,\bar{u}_{x^3},\bar{u}_{x^3t})dxdt-2\int_0^{t_0}\int_a^{x_0}(x-x_0)^2e^{2(t-t_0)}F(\bar{u},\bar{u}_x,\bar{u}_t,\bar{u}_{x^3},\bar{u}_{x^3t})dxdt$$

$$=-2e^{-2t_0}[h(x_0)-h(a)]-2(a-x)e^{-2t_0}h'(a)+(a-x_0)^2h''(a)$$

$$-(a-x_0)g'(t_0)+\frac{1}{2}(a-x_0)^2\varphi'(t_0)-u_t(x_0,t_0)+f'(t_0)$$

$$-\frac{1}{2}(a-x_0)^2\varphi(t_0)+(a-x_0)^2\int_0^{t_0}e^{2(t-t_0)}\varphi(t)dt$$

$$+(a-x_0)g(t_0)-2\int_0^{t_0}e^{2(t-t_0)}g(t)dt$$

$$+u(x_0,t_0)-f(t_0)-2\int_0^{t_0}e^{2(t-t_0)}[u(x,t)-f(t)]dt. \tag{1.7}$$

(1.6)×2+(1.7)，得

$$\int_a^{x_0}(x-x_0)^2F(\bar{u},\bar{u}_x,\bar{u}_t,\bar{u}_{x^3},\bar{u}_{x^3t})dx$$

$$=-u_t(x_0,t_0)-u(x_0,t_0)+f'(t_0)+f(t_0)-(a-x_0)g'(t_0)-(a-x_0)g(t_0)$$

$$+\frac{1}{2}(a-x_0)^2\varphi'(t_0)+\frac{1}{2}(a-x_0)^2\varphi(t_0). \tag{1.8}$$

(1.8)两端关于 t_0 积分并利用 $u(x,0)=h(x)$，以整理得

$$u(x_0,t_0)=h(x_0)e^{-t_0}+[f(t_0)-e^{-t_0}f(0)]-(a-x_0)[g(t_0)-e^{-t_0}g(0)]$$

$$+\frac{1}{2}(a-x_0)^2[\varphi(t_0)-e^{-t_0}\varphi(0)]$$

$$-\int_0^{t_0}\int_a^{x_0}(x-x_0)^2e^{t-t_0}F(\bar{u},\bar{u}_x,\bar{u}_t,\bar{u}_{x^3},\bar{u}_{x^3t})dxdt. \tag{1.9}$$

上述结果表明，若辅助问题(T″)有正则解 u，则它必满足公式(1.9). 反之，由公式(1.9)给出的函数 u，不难验证，它必为问题(T″)的正则解.

于是有如下引理.

引理 特征问题(T′)有正则解的充分必要条件是积分微分方程

$$u(x_0,t_0)=e^{-t_0}h(x_0)+[f(t_0)-e^{-t_0}f(0)]-(a-x_0)[g(t_0)-e^{-t_0}g(0)]$$

$$+\frac{1}{2}(a-x_0)^2[\varphi(t_0)-\varphi(0)]$$

$$-\int_0^{t_0}\int_a^{x_0}(x-x_0)^2e^{t-t_0}F(u,u_x,u_t,u_{x^3},u_{x^3t})dxdt \tag{1.10}$$

有解；若有解，则它们的解相同.

二、不动点

在 $C_0^4(\bar{\Omega})$ 上定义映射

$$
\begin{aligned}
[Tu](x_0,t_0) = {} & e^{-t_0}h(x_0) + [f(t_0) - e^{-t_0}f(0)] - (a - x_0)[g(t_0) - e^{-t_0}g(0)] \\
& + \frac{1}{2}(a - x_0)^2[\varphi(t_0) - e^{-t_0}\varphi(0)] \\
& - \int_0^{t_0}\int_a^{x_0}(x - x_0)^2 e^{t-t_0}F(u, u_x, u_t, u_{x^3}, u_{x^3 t})\,dx\,dt. \qquad (2.1)
\end{aligned}
$$

若能证明 T 为由 $C_0^4(\bar{\Omega})$ 到自身映射，则特征问题(T′)就化成了不动点问题.

定理 特征问题(T′)有唯一正则解 $u^* \in C_0^4(\bar{\Omega})$，且对任给的 $u_0 \in C_0^4(\bar{\Omega})$，序列

$$
\begin{aligned}
u_n(x_0,t_0) = {} & e^{-t_0}h(x_0) + [f(t_0) - e^{-t_0}f(0)] - (a - x_0)[g(t_0) - g(0)] \\
& + \frac{1}{2}(a - x_0)^2[\varphi(t_0) - e^{-t_0}\varphi(0)] \\
& - \int_0^{t_0}\int_a^{x_0}(x - x_0)^2 e^{t-t_0}F(u_{n-1}, u_{n-1\,x}, u_{n-1\,t}, u_{n-1\,x^3}, u_{n-1\,x^3 t})\,dx\,dt \qquad (2.2)
\end{aligned}
$$

于 $\bar{\Omega}$ 上一致收敛于 u^*.

证明 对 $\forall\ u, \bar{u} \in C_0^4(\bar{\Omega})$ 由于

$$
\begin{aligned}
1°)\ & \left|\frac{u_t}{(1+u^2)(1+u_x^2)(1+u_t^2)(1+u_{x^3}^2)} - \frac{\bar{u}_t}{(1+\bar{u}^2)(1+\bar{u}_x^2)(1+\bar{u}_t^2)(1+\bar{u}_{x^3}^2)}\right| \\
\leqslant {} & \frac{1}{(1+u^2)(1+u_x^2)(1+u_{x^3}^2)}\left|\frac{u_t}{1+u_t^2} - \frac{u_t}{1+\bar{u}_t^2}\right| \\
& + \frac{|u_t|}{1+\bar{u}_t^2}\left|\frac{1}{(1+u^2)(1+u_x^2)(1+u_{x^3}^2)} - \frac{1}{(1+\bar{u}^2)(1+\bar{u}_x^2)(1+\bar{u}_{x^3}^2)}\right| \\
\leqslant {} & |u_t - \bar{u}_t| + \frac{|u_t||\bar{u}_t||u_t - \bar{u}_t|}{(1+u^2)(1+u_x^2)(1+u_{x^3}^2)(1+u_t^2)(1+\bar{u}_t^2)} \\
& + \frac{|\bar{u}_{x^3}|}{1+\bar{u}_{x^3}^2}\left\{\frac{1}{1+u^2}\left|\frac{1}{1+u_x^2} - \frac{1}{1+\bar{u}_x^2}\right| + \frac{1}{1+\bar{u}_x^2}\left|\frac{1}{1+u^2} - \frac{1}{1+\bar{u}^2}\right|\right\} \\
& + \frac{1}{(1+u^2)(1+u_x^2)}\left|\frac{u_{x^3}}{1+u_{x^3}^2} - \frac{\bar{u}_{x^3}}{1+\bar{u}_{x^3}^2}\right| \\
< {} & \frac{5}{4}(|u - \bar{u}| + |u_x - \bar{u}_x| + |u_t - \bar{u}_t| + |u_{x^3} - \bar{u}_{x^3}|); \qquad (2.3)
\end{aligned}
$$

$$
\begin{aligned}
2°)\ & \left|\frac{auu_x}{(1+u^2)(1+u_x^2)(1+u_t^2)(1+u_{x^3}^2)} - \frac{a\bar{u}\bar{u}_x}{(1+\bar{u}^2)(1+\bar{u}_x^2)(1+\bar{u}_t^2)(1+\bar{u}_{x^3}^2)}\right| \\
\leqslant {} & \frac{|a||u|}{(1+u^2)(1+u_t^2)(1+u_{x^3}^2)}\left|\frac{u_x}{1+u_x^2} - \frac{\bar{u}_x}{1+\bar{u}_x^2}\right| \\
& + \frac{|a||u_x|}{1+\bar{u}_x^2}\left|\frac{u}{(1+u^2)(1+u_x^2)(1+u_{x^3}^2)} - \frac{\bar{u}}{(1+\bar{u}^2)(1+\bar{u}_t^2)(1+\bar{u}_{x^3}^2)}\right| \\
\leqslant {} & \frac{5|a|}{8}|u_x - \bar{u}_x| + \frac{|a|}{2}\frac{1}{(1+u_x^2)(1+u_{x^3}^2)}\left|\frac{u}{1+u^2} - \frac{\bar{u}}{1+\bar{u}^2}\right| \\
& + \frac{|a|}{2}\left[\frac{1}{1+u_t^2}\left|\frac{1}{1+u_{x^3}^2} - \frac{1}{1+\bar{u}_{x^3}^2}\right| + \frac{1}{1+\bar{u}_{x^3}^2}\left|\frac{1}{1+u_t} - \frac{1}{1+\bar{u}_t^2}\right|\right] \\
< {} & \frac{5|a|}{8}(|u - \bar{u}| + |u_x - \bar{u}_x| + |u_t - \bar{u}_t| + |u_{x^3} - \bar{u}_{x^3}|); \qquad (2.4)
\end{aligned}
$$

$$3°)\ \left|\frac{\varepsilon u^2u_x}{(1+u^2)(1+u_x^2)(1+u_t^2)(1+u_{x^3}^2)}-\frac{\varepsilon\bar{u}^2\bar{u}_x}{(1+\bar{u}^2)(1+\bar{u}_x^2)(1+\bar{u}_t^2)(1+\bar{u}_{x^3}^2)}\right|$$

$$\leqslant\frac{|\varepsilon||u^2|}{(1+u^2)(1+u_t^2)(1+u_{x^3}^2)}\left|\frac{u_x}{1+u_x^2}-\frac{\bar{u}_x}{1+\bar{u}_x^2}\right|$$

$$+\frac{|\varepsilon||\bar{u}_x|}{1+\bar{u}_x^2}\left|\frac{u^2}{(1+u_t^2)(1+u^2)(1+u_{x^3}^2)}-\frac{\bar{u}^2}{(1+\bar{u}^2)(1+\bar{u}_t^2)(1+\bar{u}_{x^3}^2)}\right|$$

$$\leqslant\frac{5|\varepsilon|}{4}|u_x-\bar{u}_x|+\frac{|\varepsilon|}{2}\{\frac{1}{(1+u_t^2)(1+u_{x^3}^2)}\left|\frac{u^2}{1+u^2}-\frac{\bar{u}^2}{1+\bar{u}^2}\right|$$

$$+\frac{|\bar{u}|^2}{1+\bar{u}^2}\left|\frac{1}{(1+u_t^2)(1+u_{x^3}^2)}-\frac{1}{(1+\bar{u}_t^2)(1+\bar{u}_{x^3}^2)}\right|$$

$$<\frac{5|\varepsilon|}{4}(|u-\bar{u}|+|u_x-\bar{u}_x|+|u_t-\bar{u}_t|+|u_{x^3}-\bar{u}_{x^3}|). \qquad (2.5)$$

4°) 同理

$$\left|\frac{\mu u_{x^3}}{(1+u^2)(1+u_x^2)(1+u_t^2)(1+u_{x^3}^2)}-\frac{\mu\bar{u}_{x^3}}{(1+\bar{u}^2)(1+\bar{u}_x^2)(1+\bar{u}_t^2)(1+\bar{u}_{x^3}^2)}\right|$$

$$<\frac{5|\mu|}{4}(|u-\bar{u}|+|u_x-\bar{u}_x|+|u_t-\bar{u}_t|+|u_{x^3}-\bar{u}_{x^3}|). \qquad (2.6)$$

所以

$$|F(u,u_x,u_t,u_{x^3},u_{x^3t})-F(\bar{u},\bar{u}_x,\bar{u}_t,\bar{u}_{x^3},\bar{u}_{x^3t})|$$

$$=\frac{1}{2}\left|\frac{u_t+\alpha uu_x+\varepsilon u^2u_x+\mu u_{x^3}}{\beta(1+u^2)(1+u_x^2)(1+u_t^2)(1+u_{x^3}^2)}\right.$$

$$\left.-\frac{\bar{u}_t+\alpha\bar{u}\bar{u}_x+\varepsilon\bar{u}^2\bar{u}_x+\mu\bar{u}_{x^3}}{\beta(1+\bar{u}^2)(1+\bar{u}_x^2)(1+\bar{u}_t^2)(1+\bar{u}_{x^3}^2)}\right|-(\sigma-1)|(u_{x^3t}-\bar{u}_{x^3t})|$$

$$<\frac{5}{8|\beta|}(1+\frac{|\alpha|}{2}+|\varepsilon|+|\mu|)(|u-\bar{u}|+|u_x-\bar{u}_x|+|u_t-\bar{u}_t|+|u_{x^3}-\bar{u}_{x^3})$$

$$+\frac{\sigma-1}{2}|u_{x^3t}-\bar{u}_{x^3t}|$$

$$<\frac{\sigma-1}{2}(|u-\bar{u}|+|u_x-\bar{u}_x|+|u_t-\bar{u}_t|+|u_{x^3}-\bar{u}_{x^3}|+|u_{x^3t}-\bar{u}_{x^3t}).$$

由

$$|[Tu]-[T\bar{u}]|(x_0,t_0)<\frac{(\sigma-1)(b-a)^2}{2\lambda^2}\|u-\bar{u}\|_\lambda\exp[\lambda(x_0+t_0)],$$

$$\frac{\partial}{\partial x_0}|[Tu]-[T\bar{u}]|(x_0,t_0)<\frac{(\sigma-1)(b-a)^2}{\lambda^2}\|u-\bar{u}\|_\lambda\exp[\lambda(x_0+t_0)],$$

$$\frac{\partial}{\partial t_0}|[Tu]-[T\bar{u}]|(x_0,t_0)<\frac{(\sigma-1)}{2}[\frac{(b-a)^2}{\lambda}+\frac{(b-a)^2}{\lambda^2}]\|u-\bar{u}\|_\lambda\exp[\lambda(x_0+t_0)],$$

$$\frac{\partial^3}{\partial x_0^3}|[Tu]-[T\bar{u}]|(x_0,t_0)<\frac{(\sigma-1)}{\lambda}\|u-\bar{u}\|_\lambda\exp[\lambda(x_0+t_0)],$$

$$\frac{\partial^4}{\partial x_0^3\partial t_0}|[Tu]-[T\bar{u}]|(x_0,t_0)<[\frac{\sigma-1}{\lambda}+\sigma-1]\|u-\bar{u}\|_\lambda\exp[\lambda(x_0+t_0)].$$

于是 $\|Tu-T\bar{u}\|_\lambda<(\sigma-1)[\frac{2(b-a)^2}{\lambda^2}+\frac{(b-a)^2}{2\lambda}+\frac{2}{\lambda}+1]\|u-\bar{u}\|_\lambda$，可以恰当地选择 λ，使

$$(\sigma-1)[\frac{2(b-a)^2}{\lambda^2}+\frac{(b-a^2)}{2\lambda}+\frac{2}{\lambda}+1]<1,$$

则 T 为由 $C_0^4(\bar{\Omega})$ 到自身的映射，依不动点原理，可知映射 T 有唯一的不动点 $u^* \in C_0^4(\bar{\Omega})$，即积分微分方程(1.10)有唯一的正则解 u^*. 依引理，特征问题(T′)有唯一正则解 u^*，且积分微分方程序列(2.2)于 $\bar{\Omega}$ 上一致收敛于 u^*.

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On the Characteristic Problem of the Revised KdV Equation

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Abstract

[5] studied the solution of isolated wave of the revised KdV equation $u_t + \alpha u u_x + a u^2 u_x + \mu u_{x^3} = 0$. For the characteristic problem of revised KdV equation. This paper designs a structure and gets an integral differential equation, which is equivalent to the revised KdV equation by means of Green formula and the conditions of definite solution.

Using the principle of the fixed point and the sequence of the integral differential equation, we get the iteration solution of uniform converge of the characteristic problem of the revised Kdv equation on $\bar{\Omega}$.