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非线性时滞反应扩散方程组的奇摄动*

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摘 要

本文研究了一类奇摄动非线性时滞反应扩散方程组,利用微分不等式方法,得到了解的一致有效的渐近展开式.

近来,国内、外许多学者利用微分不等式理论来广泛地讨论了常微分方程奇摄动问题(例如文献[1]-[4],[7]-[9]),特别大量地表现在非线性常微分方程(组)边值问题解的存在性及其边界层、内层性质和它的渐近性态等方面的研究.然而,利用微分不等式理论来研究偏微分方程,特别是非线性偏微分方程有关问题还不多.作者曾在文[10]中讨论了一类半线性椭圆型方程 Dirichlet 问题的奇摄动,在文[11]中讨论了一类非线性反应扩散方程组的奇摄动.本文是研究一类非线性时滞反应扩散方程组的奇摄动,得到了相应问题解的一致有效的渐近展开式.

考虑如下一个时滞耦合反应扩散方程组的奇摄动定解问题:

$$L_1[u_1] \equiv \varepsilon(u_1)_t + (a_1+b_1)u_1 = a_1u_2 + f(v_1(x,t-\varepsilon r_1)),\quad x\in\Gamma_1, 0<t\leqslant T, \tag{1}$$

$$\mathscr{L}_1[v_1] \equiv \varepsilon(v_1)_t + (a_2+b_2)v_1 = a_2v_2\quad x\in\Gamma_1, 0<t\leqslant T, \tag{2}$$

$$L_2[u_2] \equiv \varepsilon(u_2)_t - D_1\Delta u_2 + b_1u_2 = 0,\quad x\in\Omega, 0<t\leqslant T, \tag{3}$$

$$\mathscr{L}_2[v_2] \equiv \varepsilon(v_2)_t - D_2\Delta v_2 + b_2v_2 = g(u_2(x,t-\varepsilon r_2)),\quad x\in\Omega, 0<t\leqslant T, \tag{4}$$

$$B_1[u_2] \equiv \frac{\partial u_2}{\partial v} + \bar\beta_1u_2 = \bar\beta_1u_1,\quad x\in\Gamma_1, 0<t\leqslant T, \tag{5}$$

$$B_2[v_2] \equiv \frac{\partial v_2}{\partial v} + \bar\beta_2v_2 = \bar\beta_2v_1,\quad x\in\Gamma_1, 0<t\leqslant T, \tag{6}$$

$$\frac{\partial u_2}{\partial v} = \frac{\partial v_2}{\partial v} = 0,\quad x\in\Gamma_2, 0<t\leqslant T, \tag{7}$$

$$u_1(x,0,\varepsilon) = \varphi_1(x,\varepsilon),\ v_1(x,t,\varepsilon) = \psi_1(x,t,\varepsilon),\quad x\in\Gamma_1, -\varepsilon r_1\leqslant t\leqslant 0, \tag{8}$$

$$u_2(x,t,\varepsilon) = \varphi_2(x,t,\varepsilon),\ v_2(x,0,\varepsilon) = \psi_2(x,\varepsilon),\quad x\in\Omega, -\varepsilon r_2\leqslant t\leqslant 0, \tag{9}$$

其中 ε 为正的小参数,Δ 为 Laplace 算子,(u_i,v_i), $i=1,2$,表示两组化学物的密度,a_i,b_i 为与反应速度有关的正常数,$\frac{1}{\varepsilon}D_i$ 为扩散系数,$r_i\geqslant 0$ 为时滞系数,$\bar\beta_i$ 为正常数,Ω 为 R^n 中具有光滑边界的有界区域,$\partial\Omega=\Gamma_1+\Gamma_2$,$\frac{\partial}{\partial v}$为 $\partial\Omega$ 上的外法向导数,衔接条件自然需满足:

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$$\frac{\partial \varphi_2(x,0,\varepsilon)}{\partial v}+\bar{\beta}_1(\varphi_2(x,0,\varepsilon)-\varphi_1(x,\varepsilon))=0,\quad x\in\Gamma_1,$$

$$\frac{\partial \psi_2(x,\varepsilon)}{\partial v}+\bar{\beta}_2(\psi_2(x,\varepsilon)-\psi_1(x,0,\varepsilon))=0,\quad x\in\Gamma_1,$$

$$\frac{\partial \varphi_2(x,0,\varepsilon)}{\partial v}=\frac{\partial \psi_2(x,\varepsilon)}{\partial v}=0,\quad x\in\Gamma_2.$$

问题(1)－(9)是一类生化反应问题中的一个数学模型.

首先假设：

(H_1) f,g,φ_i,ψ_i 在感兴趣的区域上为充分光滑的非负函数，且存在正常数 f_0,g_0，满足：

$$\frac{\partial f}{\partial v_1}\leqslant -f_0<0,\quad 0\leqslant\frac{\partial g}{\partial u_2}\leqslant g_0.$$

显然，在假设(H_1)下，问题(1)－(9)存在唯一的一组非负解$(u_1,v_1;u_2,v_2)$. 下面我们来研究它的渐近性态.

先讨论稳态过程：

$$-D_1\Delta u_2+b_1u_2=0,\quad x\in\Omega,\tag{10}$$

$$-D_2\Delta v_2+b_2v_2=g(u_2),\quad x\in\Omega,\tag{11}$$

$$\bar{B}_1[u_2]\equiv\frac{\partial u_2}{\partial v}+\frac{\bar{\beta}_1b_1}{a_1+b_1}u_2=\frac{\bar{\beta}_1}{a_1+b_1}f(\frac{a_2v_2}{a_2+b_2}),\quad x\in\Gamma_1,\tag{12}$$

$$\bar{B}_2[v_2]\equiv\frac{\partial v_2}{\partial v}+\frac{\bar{\beta}_2b_2}{a_2+b_2}v_2=0,\quad x\in\Gamma_1,\tag{13}$$

$$\frac{\partial u_2}{\partial v}=\frac{\partial v_2}{\partial v},\quad x\in\Gamma_2,\tag{14}$$

$$u_1=\frac{a_1}{a_1+b_1}u_2+\frac{1}{a_1+b_1}f(\frac{a_2v_2}{a_2+b_2}),\quad x\in\Gamma_1,\tag{15}$$

$$v_1=\frac{a_2}{a_2+b_2}v_2,\quad x\in\Gamma_1,\tag{16}$$

设(10)－(16)的解(U_i,V_i)为：

$$U_i\sim\sum_{j=0}^{\infty}U_{ij}(x)\varepsilon^j,\quad i=1,2,\tag{17}$$

$$V_i\sim\sum_{j=0}^{\infty}V_{ij}(x)\varepsilon^j,\quad i=1,2.\tag{18}$$

展开 f,g：

$$f(\frac{a_2V_2}{a_2+b_2})\equiv F(\varepsilon)\sim\sum_{j=0}^{\infty}F_j\varepsilon^j,\tag{19}$$

$$g(U_2)\equiv G(\varepsilon)\sim\sum_{j=0}^{\infty}G_j\varepsilon^j,\tag{20}$$

其中

$$F_0=f(\frac{a_2V_{20}}{a_2+b_2}),\quad G_0=g(U_{20}),$$

$$F_j=\frac{1}{j!}\frac{\partial^j F}{\partial\varepsilon^j}\Big|_{\varepsilon=0}=\frac{a_2}{(a_2+b_2)}f_{v_2}(\frac{a_2V_{20}}{a_2+b_2})V_{2j}+f_j,\quad j=1,2,\cdots,$$

$$G_j=\frac{1}{j!}\frac{\partial^j G}{\partial\varepsilon^j}\Big|_{\varepsilon=0}=g_{u_2}(U_{20})U_{2j}+g_j,\quad j=1,2,\cdots.$$

上述 f_j, g_j 为逐次已知的函数，为简缩书写，其结构从略.

将(17)$_2$，(18)$_2$，(19)，(20)代入(10)－(14)，合并 ε 的同次幂项，并令 ε^j，$j=0,1,2,\cdots$，的系数为零，得：

$$-D_1\Delta U_{2j}+b_1U_{2j}=0,\quad x\in\Omega, \tag{21$_j$}$$

$$-D_2\Delta V_{2j}+b_2V_{2j}=g_j,\quad x\in\Omega, \tag{22$_j$}$$

$$\overline{B}_1[U_{2j}]=\frac{\overline{\beta}_1}{a_1+b_1}F_j,\ \overline{B}_2[V_{2j}]=0,\quad x\in\Gamma_1, \tag{23$_j$}$$

$$\frac{\partial U_{2j}}{\partial v}=\frac{\partial V_{2j}}{\partial v}=0,\quad x\in\Gamma_2. \tag{24$_j$}$$

现在再假设：

(H_2)　定解问题(21)$_0$－(24)$_0$存在唯一的一组足够光滑的正解(U_{20}, V_{20}).

事实上，仅需对已知函数 f, g 再赋予一定的界定条件，假设(H_2)是可行的.

依次地唯一确定线性问题(21)$_j$－(24)$_j$的解(U_{2j}, V_{2j})，$j=1,2,\cdots$. 将它们代入(17)$_2$，(18)$_2$，并形式地完全确定了(U_2, V_2)，再由(15)，(16)，便可相应确定(17)$_1$，(18)$_1$中的 U_1, V_1.

显然，由上述方法确定的(U_i, V_i)，$i=1,2$，满足(1)－(7)，但未必满足(8)，(9)，为此，对于原问题(1)－(9)的解(u_i, v_i)，还需附加校正项($\overline{U}_i, \overline{V}_i$).

今引入伸长变量[5]：

$$\tau=\frac{t}{\varepsilon},\quad 0<\varepsilon\ll 1.$$

并设

$$u_i=U_i+\overline{U}_i(x,\tau,\varepsilon),\quad i=1,2,$$
$$v_i=V_i+\overline{V}_i(x,\tau,\varepsilon),\quad i=1,2.$$

将其代入(1)－(9)，得：

$$\begin{aligned}\overline{L}_1[\overline{U}_1]&\equiv(\overline{U}_1)_\tau+(a_1+b_1)\overline{U}_1\\&=a_1\overline{U}_2+f(V_1+\overline{V}_1(x,\tau-r_1,\varepsilon))-f(V_1),\quad x\in\Gamma_1,\ \tau>0,\end{aligned} \tag{25}$$

$$\overline{\mathscr{L}}_1[\overline{V}_1]\equiv(\overline{V}_1)_\tau+(a_2+b_2)\overline{V}_1=a_2\overline{V}_2,\quad x\in\Gamma_1,\ \tau>0, \tag{26}$$

$$\overline{L}_2[\overline{U}_2]\equiv(\overline{U}_2)_\tau-D_1\Delta\overline{U}_2+b_1\overline{U}_2=0,\quad x\in\Omega,\ \tau>0, \tag{27}$$

$$\begin{aligned}\overline{\mathscr{L}}_2[\overline{V}_2]&\equiv(\overline{V}_2)_\tau-D_2\Delta\overline{V}_2+b_2\overline{V}_2\\&=g(U_2+\overline{U}_2(x,\tau-r_2,\varepsilon))-g(U_2),\quad x\in\Omega,\ \tau>0,\end{aligned} \tag{28}$$

$$B_1[\overline{U}_2]=\overline{\beta}_1\overline{U}_1,\ B_2[\overline{V}_2]=\overline{\beta}_2\overline{V}_1,\quad x\in\Gamma_1,\ \tau>0, \tag{29}$$

$$\frac{\partial\overline{U}_2}{\partial v}=\frac{\partial\overline{V}_2}{\partial v}=0,\quad x\in\Gamma_2,\ \tau>0, \tag{30}$$

$$\left.\begin{aligned}\overline{U}_1(x,0,\varepsilon)&=\varphi_1(x,\varepsilon)-U_1(x,\varepsilon),\\\overline{V}_1(x,\tau,\varepsilon)&=\psi_1(x,\varepsilon\tau,\varepsilon)-V_1(x,\varepsilon),\end{aligned}\right.\quad x\in\Gamma_1,\ -r_1\leqslant\tau\leqslant 0, \tag{31}$$

$$\left.\begin{aligned}\overline{U}_2(x,\tau,\varepsilon)&=\varphi_2(x,\varepsilon\tau,\varepsilon)-U_2(x,\varepsilon),\\\overline{V}_2(x,0,\varepsilon)&=\psi_2(x,\varepsilon)-V_2(x,\varepsilon),\end{aligned}\right.\quad x\in\Omega,\ -r_2\leqslant\tau\leqslant 0, \tag{32}$$

令

$$\overline{U}_i\sim\sum_{j=0}^{\infty}\overline{U}_{ij}(x,\tau)\varepsilon^j,\quad i=1,2, \tag{33}$$

$$\overline{V}_i \sim \sum_{j=0}^{\infty} \overline{V}_{ij}(x,\tau)\varepsilon^j, \quad i = 1,2, \tag{34}$$

且设

$$f(V_1 + \overline{V}_1(x,\tau - r_1,\varepsilon)) - f(V_1) \equiv \overline{F}(\varepsilon) \sim \sum_{j=0}^{\infty} \overline{F}_j \varepsilon^j, \tag{35}$$

$$g(U_2 + \overline{U}_2(x,\tau - r_2,\varepsilon)) - g(U_2) \equiv \overline{G}(\varepsilon) \sim \sum_{j=0}^{\infty} \overline{G}_j \varepsilon^j, \tag{36}$$

$$\varphi_i \equiv \Phi_i(\varepsilon) \sim \sum_{j=0}^{\infty} \Phi_{ij}\varepsilon^j, \ \psi_i \equiv \Psi_i(\varepsilon) \sim \sum_{j=0}^{\infty} \Psi_{ij}\varepsilon^j, \quad i = 1,2, \tag{37}$$

其中

$$\begin{aligned}
\overline{F}_0 &= f(V_{10}(x) + \overline{V}_{10}(x,\tau - r_1)) - f(V_{10}(x)) \\
&= f_{v_1}(V_{10}(x) + \theta_1 \overline{V}_{10}(x,\tau - r_1))\overline{V}_{10}(x,\tau - r_1), \quad 0 < \theta_1 < 1, \\
\overline{G}_0 &= g(U_{20}(x) + \overline{U}_{20}(x,\tau - r_2)) - g(U_{20}(x)) \\
&= g_{u_2}(U_{20}(x) + \theta_2 \overline{U}_{20}(x,\tau - r_2))\overline{U}_{20}(x,\tau - r_2), \quad 0 < \theta_2 < 1, \\
\overline{F}_j &= \frac{1}{j!}\frac{\partial^j \overline{F}}{\partial \varepsilon^j}\Big|_{\varepsilon=0} = f_{v_1}(V_{10}(x) + \overline{V}_{10}(x,\tau - r_1))\overline{V}_{1j}(x,\tau - r_1) + \bar{f}_i, \quad j = 1,2,\cdots, \\
\overline{G}_j &= \frac{1}{j!}\frac{\partial^j \overline{G}}{\partial \varepsilon^j}\Big|_{\varepsilon=0} = g_{u_2}(U_{20}(x) + \overline{U}_{20}(x,\tau - r_2))\overline{U}_{2j}(x,\tau - r_2) + \bar{g}_j, \quad j = 1,2,\cdots, \\
\Phi_{ij} &= \frac{1}{j!}\frac{\partial^j \Phi_i}{\partial \varepsilon^j}\Big|_{\varepsilon=0}, \Psi_{ij} = \frac{1}{j!}\frac{\partial^j \Psi_i}{\partial \varepsilon^j}\Big|_{\varepsilon=0}, \quad i = 1,2;\ j = 0,1,2,\cdots.
\end{aligned}$$

上述 $\bar{f}_j, \bar{g}_j$ 也为逐次地已知，为简缩书写，其结构从略.

将(33)—(37)代入(25)—(32)，合并 ε 的同次幂项，并令 ε^j，$j=0,1,2,\cdots$，的系数为零，得：

$$\overline{L}_1[\overline{U}_{1j}] = a_1\overline{U}_{2j} + \overline{F}_j,\ \overline{\mathscr{L}}_1[\overline{V}_{1j}] = a_2\overline{V}_{2j}, \quad x \in \Gamma_1,\ \tau > 0, \tag{38$_j$}$$

$$\overline{L}_2[\overline{U}_{2j}] = 0,\ \overline{\mathscr{L}}_2[\overline{V}_{2j}] = \overline{G}_j, \quad x \in \Omega,\ \tau > 0, \tag{39$_j$}$$

$$B_1[\overline{U}_{2j}] = \overline{\beta}_1\overline{U}_{1j},\ B_2[\overline{V}_{2j}] = \overline{\beta}_2\overline{V}_{1j}, \quad x \in \Gamma_1,\ \tau > 0, \tag{40$_j$}$$

$$\frac{\partial \overline{U}_{2j}}{\partial v} = \frac{\partial \overline{V}_{2j}}{\partial v} = 0, \qquad x \in \Gamma_2,\ \tau > 0, \tag{41$_j$}$$

$$\overline{U}_{1j}(x,0) = \Phi_{1j} - U_{1j},\ \overline{V}_{1j}(x,\tau) = \Psi_{1j} - V_{1j}, \quad x \in \Gamma_1,\ -r_1 \leqslant \tau \leqslant 0, \tag{42$_j$}$$

$$\overline{U}_{2j}(x,\tau) = \Phi_{2j} - U_{2j},\ \overline{V}_{2j}(x,0) = \Psi_{2j} - V_{2j}, \quad x \in \Omega,\ -r_2 \leqslant \tau \leqslant 0, \tag{43$_j$}$$

可以证明，定解问题$(38)_0$—$(43)_0$存在唯一的一组解$(\overline{U}_{10},\overline{V}_{10};\overline{U}_{20},\overline{V}_{20})$. 继而可依次地确定线性问题$(38)_j$—$(43)_j$的唯一解$(\overline{U}_{1j},\overline{V}_{1j};\overline{U}_{2j},\overline{V}_{2j})$，$j=1,2,\cdots$. 将所得到的 $\overline{U}_{ij},\overline{V}_{ij}$ 代入(33)，(34). 由此我们便得到了原问题(1)—(9)解的形式渐近展开式：

$$u_i \sim \sum_{j=0}^{\infty}(U_{ij}(x) + \overline{U}_{ij}(x,\tau))\varepsilon^j, \quad i = 1,2, \tag{44}$$

$$v_i \sim \sum_{j=0}^{\infty}(V_{ij}(x) + \overline{V}_{ij}(x,\tau))\varepsilon^j, \quad i = 1,2. \tag{45}$$

下面来进一步讨论(44)，(45)当 ε 足够小时，在感兴趣的区域上的一致有效性. 现有如下定理：

定理 在条件(H_1)，(H_2)下，定解问题(1)—(9)的解$(u_1,v_1;u_2,v_2)$在 $x\in\overline{\Omega}$，$0\leqslant t\leqslant T$ 上当 ε

足够小时，对任意的正整数 m 关于 ε 一致地成立：

$$u_i = \sum_{j=0}^{m}(U_{ij}(x) + \overline{U}_{ij}(x,\frac{t}{\varepsilon}))\varepsilon^j + O(\varepsilon^{m+1}),\quad i=1,2;\ 0<\varepsilon\ll 1, \tag{46}$$

$$v_i = \sum_{j=0}^{m}(V_{ij}(x) + \overline{V}_{ij}(x,\frac{t}{\varepsilon}))\varepsilon^j + O(\varepsilon^{m+1}),\quad i=1,2;\ 0<\varepsilon\ll 1. \tag{47}$$

证明 作辅助函数 $\alpha_i,\beta_i,\alpha_i',\beta_i'$：

$$\alpha_i(x,t,\varepsilon) = Y_{im}(x,t,\varepsilon) - \gamma_i\varepsilon^{m+1},\quad i=1,2, \tag{48}$$

$$\beta_i(x,t,\varepsilon) = Y_{im}(x,t,\varepsilon) + \gamma_i\varepsilon^{m+1},\quad i=1,2, \tag{49}$$

$$\alpha_i'(x,t,\varepsilon) = Y_{im}'(x,t,\varepsilon) - \gamma_i'\varepsilon^{m+1},\quad i=1,2, \tag{50}$$

$$\beta_i'(x,t,\varepsilon) = Y_{im}'(x,t,\varepsilon) + \gamma_i'\varepsilon^{m+1},\quad i=1,2, \tag{51}$$

其中

$$Y_{im}(x,t,\varepsilon) = \sum_{j=0}^{m}[U_{ij}(x) + \overline{U}_{ij}(x,\frac{t}{\varepsilon})]\varepsilon^j,$$

$$Y_{im}'(x,t,\varepsilon) = \sum_{j=0}^{m}[V_{ij}(x) + \overline{V}_{ij}(x,\frac{t}{\varepsilon})]\varepsilon^j,$$

而 γ_i,γ_i' 为待定正常数，将在下文中确定.

显然，当 ε 足够小时，可得：

$$\alpha_i\geqslant 0,\ \alpha_i'\geqslant 0,\ \beta_i\geqslant 0,\ \beta_i'\geqslant 0,\quad i=1,2, \tag{52}$$

$$\frac{\partial\alpha_2}{\partial v} = \frac{\partial\alpha_2'}{\partial v} = \frac{\partial\beta_2}{\partial v} = \frac{\partial\beta_2'}{\partial v} = 0,\quad x\in\Gamma_2,\ 0\leqslant t\leqslant T. \tag{53}$$

$$\begin{aligned}L_2[\alpha_2] &= -D_1\Delta(\sum_{j=0}^{m}U_{2j}\varepsilon^j) + b_1\sum_{j=0}^{m}U_{2j}\varepsilon^j + \sum_{j=0}^{m}(\overline{U}_{2j})_t\varepsilon^j - D_1\Delta(\sum_{j=0}^{m}\overline{U}_{2j}\varepsilon^j)\\ &\quad + b_1\sum_{j=0}^{m}\overline{U}_{2j}\varepsilon^j - b_1\gamma_2\varepsilon^{m+1}\\ &= \sum_{j=0}^{m}[-D_1\Delta U_{2j} + b_1U_{2j}]\varepsilon^j + \sum_{j=0}^{m}\overline{L}_2[\overline{U}_{2j}]\varepsilon^j - b_1\gamma_2\varepsilon^{m+1}\\ &= -b_1\gamma_2\varepsilon^{m+1}<0,\quad x\in\Omega,\ 0<t\leqslant T.\end{aligned} \tag{54}$$

同理有

$$L_2[\beta_2]>0,\quad x\in\Omega,\ 0<t\leqslant T. \tag{55}$$

又不难看出，存在正常数 M_i，$i=1,2,\cdots,6$，有：

$$\alpha_1(x,0,\varepsilon)\leqslant\varphi_1(x,\varepsilon) - (\gamma_1 - M_1)\varepsilon^{m+1},\quad x\in\Gamma_1,$$

$$\alpha_1'(x,t,\varepsilon)\leqslant\psi_1(x,t,\varepsilon) - (\gamma_1' - M_2)\varepsilon^{m+1},\quad x\in\Gamma_1,\ -\varepsilon\tau_1\leqslant t\leqslant 0,$$

$$\alpha_2(x,t,\varepsilon)\leqslant\varphi_2(x,t,\varepsilon) - (\gamma_2 - M_3)\varepsilon^{m+1},\quad x\in\Omega,\ -\varepsilon\tau_2\leqslant t\leqslant 0,$$

$$\alpha_2'(x,0,\varepsilon)\leqslant\psi_2(x,\varepsilon) - (\gamma_2' - M_4)\varepsilon^{m+1},\quad x\in\Omega,$$

$$B_1[\alpha_2] - \overline{\beta}_1\alpha_1 = -\overline{\beta}_1(\gamma_2 - \gamma_1)\varepsilon^{m+1},\quad x\in\Gamma_1,\ t>0,$$

$$B_2[\alpha_2'] - \overline{\beta}_1\alpha_1' = -\overline{\beta}_2(\gamma_2' - \gamma_1')\varepsilon^{m+1},\quad x\in\Gamma_1,\ t>0,$$

和

$$L_1[\alpha_1] - a_1\alpha_2 - f(\alpha_1'(x,t-\varepsilon\tau_1))\leqslant\sum_{j=0}^{m}[(a_1+b_1)U_{1j} - a_1U_{2j} - F_j]\varepsilon^j$$

$$+\sum_{j=0}^{m}[\overline{L}[\overline{U}_{ij}]-a_1\overline{U}_{2j}-\overline{F}_j]\varepsilon^j-(a_1+b_1)\gamma_1\varepsilon^{m+1}+a_1\gamma_2\varepsilon^{m+1}+M_5\varepsilon^{m+1}$$

$$+[f(Y_{1m}'(x,t-\varepsilon r_1))-f(\alpha_1'(x,t-\varepsilon r_1))]$$

$$\leqslant-(f_0\gamma_1'+(a_1+b_1)\gamma_1-a_1\gamma_2-M_5)\varepsilon^{m+1},\quad x\in\Gamma_1,\ 0<t\leqslant T.$$

$$\mathscr{L}_1[\alpha_1']-a_2\alpha_2'=\sum_{j=0}^{m}[(a_2+b_2)V_{1j}-a_2V_{2j}]\varepsilon^j$$

$$+\sum_{j=0}^{m}[\overline{\mathscr{L}}_1[\overline{V}_{1j}]-a_2\overline{V}_{2j}]\varepsilon^j-(a_2+b_2)\gamma_1'\varepsilon^{m+1}+a_2\gamma_2'\varepsilon^{m+1}$$

$$=-((a_2+b_2)\gamma_1'-a_2\gamma_2')\varepsilon^{m+1},\quad x\in\Gamma_1,\ 0<t\leqslant T,$$

$$\mathscr{L}_2[\alpha_2']-g(\alpha_2(x,t-\varepsilon r_2))\leqslant\sum_{j=0}^{m}[-D_2\Delta V_{2j}+b_2V_{2j}-g_j]\varepsilon^j$$

$$+\sum_{j=0}^{m}[\overline{\mathscr{L}}_2[\overline{V}_{2j}]-\overline{G}_j]\varepsilon^j-b_2\gamma_2'\varepsilon^{m+1}+M_6\varepsilon^{m+1}$$

$$+[g(Y_{2m}(x,t-\varepsilon r_2))-g(\alpha_2(x,t-\varepsilon r_2))]$$

$$\leqslant-(b_2\gamma_2'-g_0r_2-M_6)\varepsilon^{m+1},\quad x\in\Omega,\ 0<t\leqslant T.$$

同理,用 β_i,β_i' 相应地代替 α_i,α_i',可证上述相应的各式有相反的不等式.

对足够小的 ε,我们可作如下的选定:

$$\gamma_1=\gamma_2=\max\{M_i,\ i=1,2,3,4\},$$

$$\gamma_1'=\max\{\frac{1}{f_0}(-b_1\gamma_1+M_5),\frac{a_2}{b_2(a_2+b_2)}(g_0\gamma_1+M_6)\},$$

$$\gamma_2'=\frac{a_2+b_2}{a_2}\gamma_1'.$$

因此,在上述选定的 γ_i,γ_i' 下,恒有:

$$\alpha_1(x,0,\varepsilon)\leqslant\varphi_1(x,\varepsilon)\leqslant\beta_1(x,0,\varepsilon),\quad x\in\Gamma_1,\tag{56}$$

$$\alpha_1'(x,t,\varepsilon)\leqslant\psi_1(x,t,\varepsilon)\leqslant\beta_1'(x,0,t,\varepsilon),\quad x\in\Gamma_1,\ -\varepsilon r_1\leqslant t\leqslant 0,\tag{57}$$

$$\alpha_2(x,t,\varepsilon)\leqslant\varphi_2(x,t,\varepsilon)\leqslant\beta_2(x,t,\varepsilon),\quad x\in\Omega,\ -\varepsilon r_2\leqslant t\leqslant 0,\tag{58}$$

$$\alpha_2'(x,0,\varepsilon)\leqslant\psi_2(x,\varepsilon)\leqslant\beta_2'(x,0,\varepsilon),\quad x\in\Omega,\tag{59}$$

$$B_1[\alpha_2]-\overline{\beta}_1\alpha_1=0=B_1[\beta_2]-\overline{\beta}_1\beta_1,\quad x\in\Gamma_1,\ t>0,\tag{60}$$

$$B_2[\alpha_2']-\overline{\beta}_2\alpha_1'<0<B_2[\beta_2']-\overline{\beta}_2\beta_1',\quad x\in\Gamma_1,\ t>0,\tag{61}$$

$$L_1[\alpha_1]-a_1\alpha_1-f(\alpha_1'(x,t-\varepsilon r_1))\leqslant 0$$

$$\leqslant L_1[\beta_1]-a_1\beta_1-f(\beta_1'(x,t-\varepsilon r_1)),\quad x\in\Gamma_1,\ 0<t\leqslant T,\tag{62}$$

$$\mathscr{L}_1[\alpha_1']-a_2\alpha_2'=0=\mathscr{L}_1[\beta_1']-a_2\beta_2',\quad x\in\Gamma_1,\ 0<t\leqslant T,\tag{63}$$

$$L_2[\alpha_2]\leqslant 0\leqslant L_2[\beta_2],\quad x\in\Omega,0<t\leqslant T,\tag{64}$$

$$\mathscr{L}_2[\alpha_2']-g(\alpha_2(x,t-\varepsilon r_2))\leqslant 0$$

$$\leqslant\mathscr{L}_2[\beta_2']-g(\beta_2(x,t-\varepsilon r_1)),\quad x\in\Omega,\ 0<t\leqslant T,\tag{65}$$

由关系式(52)—(65)及文献[6]知,(u_i,v_i), $i=1,2$,在 $x\in\overline{\Omega}$, $0\leqslant t\leqslant T$ 上,对足够小的 ε 一致地成立:

$$\alpha_i(x,t,\varepsilon)\leqslant u_i(x,t,\varepsilon)\leqslant\beta_i(x,t,\varepsilon),\quad \alpha_i'(x,t,\varepsilon)\leqslant v_i(x,t,\varepsilon)\leqslant\beta_i'(x,t,\varepsilon).$$

由上述两个关系式及(48)—(51),我们便得到了关系式(46),(47)在 $x\in\overline{\Omega}$, $0\leqslant t\leqslant T$ 上关于 ε 一致地成立. 定理证毕.

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Singular Perturbation for the Nonlinear Time Lag Reaction Duffusion System

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Abstract

In this paper, the problem for the singularly perturbed nonlinear time lag reaction duffusion system is considered. The uniformly valid asymptotic expansion of the solution is obtained by using the differential inequalities.