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分担两个值的亚纯函数的唯一性定理

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摘 要

本文主要讨论分担两个值的亚纯函数的唯一性问题，推广和改进了M. Ozawa、仪洪勋、C.C. Yang 等人的结果.

关键词: 亚纯函数，亏值，唯一性.

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Unicity Theorems of Meromorphic Functions That Share two Values*

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Abstract In this paper, we mainly discus the unicity problems of meromorphic functions that share two values. Which generalize and improve some results of M. Ozawa, Yi Hongxun, C.C. Yang etc.

Key Words meromorphic function, difficient value, unicity.

1. Introduction and main results

In this paper, we use the usual notation of Nevanlinna theorey (see [1]). Let E denote a positive real number set with finite linear measure. The notation $S(r,f)$ denote any quantity satisfying $S(r,f)=o\{T(r,f)\}$, $(r\to\infty,\ r\notin E)$, which not necessarily be the same at each occurrence. If two meromorphic functions f and g have the same a-points with the same multiplicities, we denote it by $E(a,f)=E(a,g)$.

In 1976, M. Ozawa proved the following theorem:

Theorem A ([2]) *Let f and g be two entire functions, such that $E(1,f)=E(1,g)$. If $\delta(0,f)>0$ and 0 is a lacunary for g, then $f=g$ or $f\cdot g=1$.*

In 1990, Yi Hongxun obtained the following result:

Theorem B ([3]) *Let f and g be two meromorphic functions, such that $E(1,f)=E(1,g)$, $E(\infty,f)=E(\infty,g)$. If $N(r,\frac{1}{f})+N(r,\frac{1}{g})+2\overline{N}(r,f)<(\mu+o(1))T(r)$ $(r\notin E)$, where $T(r)=\max\{T(r,f),T(r,g)\}$, $\mu<1$. Then $f=g$ or $f\cdot g=1$.*

In this paper, we prove the following theorem, which include theorem A and theorem B.

Theorem 1 *Let f and g be two nonconstant meromorphic functions, μ and λ be two meromorphic functions, satisfying*

$$T(r,\mu)=o\{T(r,f)\},\quad T(r,\lambda)=o\{T(r,g)\},$$

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Assume that $E(\infty, f) = E(\infty, g)$, $E(\varphi, f) = E(\varphi, g)$, *where* φ *is a meromorphic function, satisfying* $T(r,\varphi) = o\{\min[T(r,f), T(r,g)]\}$ *and* $\varphi \not\equiv \mu, \varphi \not\equiv \lambda$. *If*

$$N(r, \frac{1}{f-\mu}) + N(r, \frac{1}{g-\lambda}) + 2\overline{N}(r,f) < (1-\varepsilon_0)T(r) \quad (\varepsilon_0 > 0),$$

where $T(r) = \max\{T(r,f), T(r,g)\}$. *Then*

$$\frac{f-\mu}{\varphi-\mu} = \frac{g-\lambda}{\varphi-\lambda},$$

or

$$(f-\mu)\cdot(g-\lambda) = (\varphi-\mu)\cdot(\varphi-\lambda).$$

In [4], C.C. Yang has asked:

What can be said about the relationship between two entire functions f and g if

$$E(0,f) = E(0,g), \quad E(1,f') = E(1,g')?$$

In 1991, Yi Hongxun and C.C. Yang proved following result:

Theorem C ([5]) *Let* f *and* g *be two nonconstant entire functions, If* $\delta(0,f) + \delta(0,g) > 1$ *and* $E(1,f') = E(1,g')$, *then* $f = g$ *or* $f' \cdot g' = 1$.

In this paper, we generalize and improve the result of Theorem C, and obtain the following theorem:

Theorem 2 *Let* f *and* g *be two nonconstant meromorphic functions, and*

$$E(\infty,f) = E(\infty,g), \quad E(a,f') = E(b,g'),$$

where a, b *are nonzero constants. If there exist finite comples number* c, d *such that*

$$N(r, \frac{1}{f-c}) + N(r, \frac{1}{g-d}) + 3\overline{N}(r,f) < (1-\varepsilon_0)T(r) \quad (\varepsilon_0 > 0),$$

where $T(r) = \max\{T(r,f), T(r,g)\}$. *Then* $\frac{f-c}{a} = \frac{g-d}{b}$, *or* $f' \cdot g' = ab$.

2. Some lemmas

Lemma 1 ([6]) *Let* f_1 *and* f_2 *be two nonconstant meromorphic functions,* $\alpha_1 \not\equiv 0$ *and* $\alpha_2 \not\equiv 0$ *be two meromorphic functions, satisfying*

$$T(r, \alpha_i) = o\{T(r)\}, \quad (i = 1, 2),$$

where $T(r) = \max\{T(r,f_1), T(r,f_2)\}$. If $\alpha_1 f_1 + \alpha_2 f_2 = 1$, then

$$T(r,f_1) < N(r, \frac{1}{f_1}) + N(r, \frac{1}{f_2}) + \overline{N}(r,f_1) + o\{T(r)\} \quad (r \notin E).$$

Lemma 2 ([7]) *Let $f_1, f_2, \cdots, f_n$ be linearly independent meromorphic functions, satisfying $\sum_{j=1}^{n} f_j \equiv 1$. Then*

$$T(r,f_j) < \sum_{i=1}^{n} N(r,\frac{1}{f_i}) + N(r,f_j) + N(r,D) - \sum_{i=1}^{n} N(r,\frac{1}{f_i}) + o\{T(r)\} \quad (r \notin E;\ j = 1,2,\cdots,n),$$

where $T(r) = \max\{T(r,f)\}$.

$$D = \begin{vmatrix} f_1 & f_2 & \cdots & f_n \\ f_1' & f_2' & \cdots & f_n' \\ & \cdots & \cdots & \\ f_1^{(n-1)} & f_2^{(n-1)} & \cdots & f_n^{(n-1)} \end{vmatrix}$$

Lemma 3 ([3]) *Let f_1, f_2, f_3 be three nonconstant meromorphic functions, satisfying $\sum_{j=1}^{n} f_j \equiv 1$. Let $g_1 = -\frac{f_3}{f_2}, g_2 = \frac{1}{f_2}, g_3 = -\frac{f_1}{f_2}$. If f_1, f_2, f_3 are linearly independent, then g_1, g_2, g_3 are linearly independent.*

Lemma 4 *Let f be nonconstant meromorphic function, then for arbitrary finite complex number c we have $N(r,\frac{1}{f'}) \le T(r,f') + N(r,\frac{1}{f-c}) - T(r,f) + o\{T(r,f)\}$ $(r \notin E)$.*

Proof For arbitrary finite complex number c, we note that

$$m(r,\frac{1}{f-c}) \le m(r,\frac{f'}{f-c}) + m(r,\frac{1}{f'}) = m(r,\frac{1}{f'}) + S(r,f), \tag{1}$$

by the first fundamental theorem (see [1]) we have from (1)

$$T(r,f) - N(r,\frac{1}{f-c}) \le T(r,f') - N(r,\frac{1}{f'}) + S(r,f).$$

Thus $N(r,\frac{1}{f'}) \le T(r,f') + N(r,\frac{1}{f-c}) - T(r,f) + S(r,f)$ which proves Lemma 4.

3. The proof of theorems

The proof of Theorem 1 In fact, from $E(\infty,f) = E(\infty,g)$ and $E(\varphi,f) = E(\varphi,g)$ we have

$$\frac{f-\varphi}{g-\varphi} = e^{\alpha}, \tag{2}$$

where α is an entire function. From (2) we deduce that

$$f - \mu = (g-\lambda)e^{\alpha} - (\varphi - \lambda)e^{\alpha} + (\varphi - \mu). \tag{3}$$

We dividing our argument into two cases:

Case 1 e^α is identically equal to constant, suppose that $e^\alpha = k$.

(1.1) $k = 1$, from (2) we get $f = g$. Thus $f - \mu = g - \lambda + (\lambda - \mu)$. If $\lambda \neq \mu$, then

$$\frac{f-\mu}{\lambda-\mu} - \frac{g-\lambda}{\lambda-\mu} = 1, \tag{4}$$

by Lemma 1 we have

$$\begin{aligned} T(r,f) &\leq T(r, f-\mu) + o\{T(r,f)\} \\ &< N(r, \frac{1}{f-\mu}) + N(r, \frac{1}{g-\lambda}) + \overline{N}(r,f) + o\{T(r)\}, \quad (r \notin E), \end{aligned} \tag{5}$$

where $T(r) = \max\{T(r,f), T(r,g)\}$. In the same manner, we get

$$T(r,f) < N(r, \frac{1}{f-\mu}) + N(r, \frac{1}{g-\lambda}) + \overline{N}(r,f) + o\{T(r)\}, \quad (r \notin E). \tag{6}$$

Combining that (5) and (6) we deduce that

$$T(r) < N(r, \frac{1}{f-\mu}) + N(r, \frac{1}{g-\lambda}) + \overline{N}(r,f) + o\{T(r)\} < (1-\varepsilon_0)T(r), \quad (r \notin E),$$

this is a contradiction. Hence $\lambda = \mu$, thus $\dfrac{f-\mu}{\varphi-\mu} = \dfrac{g-\lambda}{\varphi-\lambda}$.

(1.2) $k \neq 1$, $k = \dfrac{\varphi-\mu}{\varphi-\lambda}$. From (2) we have $\dfrac{f-\varphi}{g-\varphi} = \dfrac{\varphi-\mu}{\varphi-\lambda}$, i.e.,

$$\frac{f-\mu}{\varphi-\mu} = \frac{g-\lambda}{\varphi-\lambda}.$$

(1.3) $k \neq 1$ and $k \neq \dfrac{\varphi-\mu}{\varphi-\lambda}$. From (3) we have

$$\frac{f-\mu}{g-x_1} - \frac{k}{x_1}(g-\lambda) = 1, \tag{7}$$

where $x_1 = (\varphi-\mu) - k(\varphi-\lambda)$. It is easy to deduce that a contradiction by Lemma 1.

Case 2 e^α is not identically equal to constant and $e^\alpha \not\equiv \dfrac{\varphi-\mu}{\varphi-\lambda}$. Let $f_1 = \dfrac{f-\mu}{\varphi-\mu}$, $f_2 = e^\alpha \dfrac{\varphi-\lambda}{\varphi-\mu}$, $f_3 = -e^\alpha \dfrac{g-\lambda}{\varphi-\mu}$, $T_1(r) = \max\limits_{1\leq j\leq 3}\{T(r,f_j)\}$. Then from (3)

$$\sum_{j=1}^{3} f_j \equiv 1. \tag{8}$$

Assume that f_1, f_2 and f_3 are liearly independent, by Lemma 3 we get

$$g_1 = \frac{g-\lambda}{\varphi-\lambda}, \quad g_2 = e^{-\alpha}\frac{\varphi-\mu}{\varphi-\lambda}, \quad g_3 = -e^{-\alpha}\frac{g-\lambda}{\varphi-\mu}$$

are linearly independent. By Lemma 2 we have

$$\begin{aligned} T(r,f) &< N(r,\frac{1}{f-\mu})+N(r,\frac{1}{g-\lambda})+\overline{N}(r,f)+N(r,D) \\ &\quad -\sum_{j=1}^{3}N(r,f_j)+o\{T_1(r)\},\quad (r\notin E), \end{aligned} \tag{9}$$

where

$$D=\begin{vmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{vmatrix}.$$

From (8)

$$D=\begin{vmatrix} f_1 & f_2 & 1 \\ f_1' & f_2' & 0 \\ f_1'' & f_2'' & 0 \end{vmatrix}=\begin{vmatrix} f_1' & f_2' \\ f_1'' & f_2'' \end{vmatrix},$$

hence $N(r,D)\le N(r,f)+2\overline{N}(r,f)+0\{T_1(r)\}$. Noting that $E(\infty,f)=E(\infty,g)$, so

$$N(r,f)+N(r,D)-\sum_{j=1}^{3}N(r,f_j)\le 2\overline{N}(r,f)+o\{T_1(r)\}.$$

Relative to (9) we obtain

$$T(r,f)<N(r,\frac{1}{f-\mu})+N(r,\frac{1}{g-\lambda})+2\overline{N}(r,f)+o\{T_1(r)\}. \tag{10}$$

In the same manner, we have

$$T(r,g)<N(r,\frac{1}{f-\mu})+N(r,\frac{1}{g-\lambda})+2\overline{N}(r,f)+o\{T_2(r)\}, \tag{11}$$

where $T_2(r)=\max_{1\le j\le 3}\{T(r,g_j)\}$.

Combining (10) and (11) we deduce that $T(r)<(1-\varepsilon_0)\cdot T(r)$, $(r\notin E)$, this is impossible.

Which show that f_1,f_2 and f_3 are linearly dependent, i.e., there exist three constants $(c_1,c_2,c_3)\neq(0,0,0)$ such that

$$c_1f_1+c_2f_2+c_3f_3=0. \tag{12}$$

If $c_1=0$, then $c_2\neq 0$ and $c_3\neq 0$, from (12) we have $g=\frac{c_3}{c_2}\varphi+(1-\frac{c_3}{c_2})\lambda$, contradicting to given condition.

Hence $c_1\neq 0$, combining (8) and (12), we get

$$(\frac{c_2}{c_1}-1)\frac{g-\lambda}{\varphi-\mu}e^{\alpha}+(\frac{c_3}{c_1}-1)e^{\alpha}=1, \tag{13}$$

assume that $\frac{c_3}{c_1} - 1 \neq 0$. Since e^α is not identically equal to constant, so $\frac{c_2}{c_1} - 1 \neq 0$. By Lemma 1 we have

$$T(r,g) < N(r,\frac{1}{g-\lambda}) + \overline{N}(r,g) + o\{T(r)\} \quad (r \notin E). \tag{14}$$

On the other hand, by a generalization of Nevanlinna's second fundament theorem (see [1])

$$\begin{aligned} T(r,f) &< \overline{N}(r,f) + N(r,\frac{1}{f-\mu}) + N(r,\frac{1}{f-\varphi}) + S(r,f) \\ &\leq \overline{N}(r,f) + N(r,\frac{1}{f-\mu}) + N(r,\frac{1}{g-\varphi}) + S(r,f) \\ &\leq \overline{N}(r,f) + N(r,\frac{1}{f-\mu}) + T(r,g) + o\{T(r)\} \\ &< 2\overline{N}(r,f) + N(r,\frac{1}{f-\mu}) + N(r,\frac{1}{g-\lambda}) + o\{T(r)\}. \end{aligned} \tag{15}$$

Combining (14) and (15) we deduce that

$$\begin{aligned} T(r) &< 2\overline{N}(r,f) + N(r,\frac{1}{f-\mu}) + N(r,\frac{1}{g-\lambda}) + o\{T(r)\} \\ &< (1-\varepsilon_o)T(r) + 0\{T(r)\} \quad (r \notin E), \end{aligned}$$

this is also impossible. Thus $\frac{c_3}{c_1} - 1 = 0$. From (13)

$$\frac{g-\lambda}{\varphi-\mu}e^\alpha = \frac{c_1}{c_2-c_1}. \tag{16}$$

From (16) and (8) we get

$$\frac{f-\mu}{\varphi-\mu} = \frac{c_2}{c_2-c_1} - \frac{\varphi-\lambda}{\varphi-\mu}e^\alpha, \tag{17}$$

it is easy to see that $c_2 = 0$ by Lemma 1. From (16) and (17) we obtain, resprectively;

$$g-\lambda = -(\varphi-\mu)e^{-\alpha}, \tag{18}$$

and

$$f-\mu = -(\varphi-\lambda)e^\alpha, \tag{19}$$

i.e., $(f-\mu)(g-\lambda) = (\varphi-\mu)(\varphi-\lambda)$, this completes the proof of Theorem 1.

The proof of Theorem 2 From given conditions $E(\infty,f) = E(\infty,g)$, $E(a,f') = E(b,g')$. We can assume that

$$(f'-a) = (g'-b)e^\alpha. \tag{20}$$

(i) $e^\alpha \equiv k$ (constant). If $k \neq a/b$, then

$$\frac{f'}{(a-bk)} - \frac{kg'}{(a-bk)} = 1, \tag{21}$$

by Lemma 1 and Lemma 4 we get

$$\begin{aligned} T(r,f') &< N(r,\frac{1}{f'})+N(r,\frac{1}{g'})+\overline{N}(r,f)+o\{T_3(r)\} \\ &\leq T(r,f')-T(r,f)+N(r,\frac{1}{f-c})+T(r,g')-T(r,g) \\ &\quad +N(r,\frac{1}{g-d})+\overline{N}(r,f)+o\{T_3(r)\}, \quad (r\notin E), \end{aligned}$$

where $T_3(r)=\max\{T(r,f'),T(r,g')\}$. Hence

$$T(r,f)<N(r,\frac{1}{f-c})+N(r,\frac{1}{g-d})+2\overline{N}(r,f)+o\{T_3(r)\} \quad (r\notin E). \tag{22}$$

Similarly, we can obtain

$$T(r,g)<N(r,\frac{1}{f-c})+N(r,\frac{1}{g-d})+2\overline{N}(r,f)+o\{T_3(r)\} \quad (r\notin E). \tag{23}$$

Obviously, $o\{T_3(r)\}=o\{T(r)\}$. Combining (22) and (23) we deduce that

$$T(r)<(1-\varepsilon_o)T(r)+0\{T(r)\} \quad (r\notin E),$$

this is contradiction. Which show that if e^{α} equal to constant, then $e^{\alpha}=a/b$. From (20) $f'=a/bg'$. Let

$$f=\frac{a}{b}g+t, \quad (t=\text{ constant }) \tag{24}$$

So $(f-c)=\frac{a}{b}(g-d)+(t-c+\frac{a}{b}d)$. Assume that $t_1\triangleq t-c+\frac{a}{b}d\neq 0$, then by Nevanlinna's second fundament theorem we have

$$\begin{aligned} T(r,f) &\leq T(r,f-c)+o(1) \\ &< \overline{N}(r,f)+N(r,\frac{1}{f-c})+N(r,\frac{1}{(f-c)-t_1})+S(r,f) \\ &= \overline{N}(r,f)+N(r,\frac{1}{f-c})+N(r,\frac{1}{g-d})+S(r,f) \\ &< (1-\varepsilon_0)T(r)+o\{T(r)\}, \quad (r\notin E). \end{aligned} \tag{25}$$

From (24) we know that

$$T(r,g)\leq(1+o(1))T(r,f). \tag{26}$$

Combining (25) and (26) we obtain $T(r)<(1-\varepsilon_0)T(r)+o\{T(r)\}$, $(r\notin E)$, this is impossible. Hence $t_1=t-c+\frac{a}{b}d=0$, i.e., $\frac{f-c}{a}=\frac{g-d}{b}$.

(ii) $e^{\alpha}\not\equiv$ constant. Let $f_1=\frac{f'}{a}$, $f_2=\frac{g'}{a}\cdot e^{\alpha}$, $f_3=\frac{b}{a}e^{\alpha}$, $T_4(r)=\max_{1\leq j\leq 3}\{T(r,f_j)\}$. From (20) we deduce that

$$\sum_{j=1}^{3}f_j\equiv 1. \tag{27}$$

Assume that f_1, f_2 and f_3 are liearly independent, by Lemma 2 we have

$$T(r,f') < N(r,\frac{1}{f'}) + N(r,\frac{1}{g'}) + \overline{N}(r,f') + N(r,D) - \sum_{j=1}^{3} N(r,f_j) + o\{T_4(r)\}, \quad (r \notin E), \tag{28}$$

where

$$D = \begin{vmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{vmatrix}.$$

From (27), we can get

$$D = \begin{vmatrix} f_1 & 1 & f_3 \\ f_1' & 0 & f_3' \\ f_1'' & 0 & f_3'' \end{vmatrix} = -\frac{b}{a^2} \begin{vmatrix} f'' & \alpha' e^{\alpha} \\ f''' & (\alpha'^2 + \alpha'')e^{\alpha} \end{vmatrix},$$

So $N(r,D) \le N(r,f') + 2\overline{N}(r,f)$. Noting that $E(\infty,f) = E(\infty,g)$, hence

$$N(r,f') + N(r,D) - \sum_{j=1}^{3} N(r,f_j) \le 2\overline{N}(r,f) + o\{T_4(r)\}.$$

From (28) and Lemma 4 we obtain

$$T(r,f) < N(r,\frac{1}{f-c}) + N(r,\frac{1}{g-d}) + 3\overline{N}(r,f) + o\{T_4(r)\}. \tag{29}$$

Next, according to Lemma 3 we know that $g_1 = \frac{g'}{b}$, $g_2 = -\frac{f'}{b} \cdot e^{-\alpha}$, $g_3 = \frac{a}{b} e^{-\alpha}$ are linearly independent. In the same manner, we can get

$$T(r,g) < N(r,\frac{1}{f-c}) + N(r,\frac{1}{g-d}) + 3\overline{N}(r,f) + o\{T_4(r)\}. \tag{30}$$

Combining that (29) and (30) we deduce that

$$T(r) < (1-\varepsilon_0) \cdot T(r) + 0\{T(r)\}, \quad (r \notin E)$$

this is impossible.

Which shows that f_1, f_2 and f_3 are linearly dependent, i.e., there exist three constants $(t_1, t_2, t_3) \ne (0,0,0)$ such that

$$t_1 f_1 + t_2 f_2 + t_3 f_3 = 0. \tag{31}$$

If $t_1 = 0$, obviously $t_2 \ne 0$ and $t_3 \ne 0$. From (31) $g' \equiv b$, from (20) we can deduce that $f' \equiv a$, hence $f' \cdot g' = ab$. If $t_1 \ne 0$, combining (20) and (31) we have

$$(\frac{t_2}{t_1} - 1)\frac{g'}{a} e^{\alpha} + (1 - \frac{t_3}{t_1}) e^{\alpha} = 1, \tag{32}$$

assume that $1-\frac{t_3}{t_1}\neq 0$, since $e^{\alpha}\not\equiv$ constant, $\frac{t_2}{t_1}-1\neq 0$, thus

$$\frac{\frac{t_2}{t_1}-1}{1-\frac{t_3}{t_1}}\cdot\frac{g'}{a}+\frac{1}{\frac{t_3}{t_1}-1}e^{-\alpha}=1,$$

by Lemma 1 and Lemma 4 we get

$$T(r,g)<N(r,\frac{1}{g-d})+\overline{N}(r,f)+o\{T(r)\},\quad (r\notin E). \tag{33}$$

On the other hand, by a generalization of Nevanlinna's second fundament theorem

$$\begin{aligned}
T(r,f) &< \overline{N}(r,f)+N(r,\frac{1}{f-c})+N(r,\frac{1}{f'-a})+S(r,f)\\
&= \overline{N}(r,f)+N(r,\frac{1}{f-c})+N(r,\frac{1}{g'-b})+S(r,f)\\
&\leq \overline{N}(r,f)+N(r,\frac{1}{f-c})+T(r,g')+o\{T(r)\}\\
&\leq N(r,\frac{1}{f-c})+2\overline{N}(r,f)+T(r,g)+o\{T(r)\}\\
&\leq N(r,\frac{1}{f-c})+3\overline{N}(r,f)+N(r,\frac{1}{g-d})+o\{T(r)\}\quad (r\notin E).
\end{aligned} \tag{34}$$

Combining (33) and (34) we deduce that $T(r)<(1-\varepsilon_0)T(r)+o\{T(r)\}$, $(r\notin E)$, this is a contradiction. So $1-t_3/t_1=0$. From (32)

$$g'=\frac{t_1}{t_2-t_1}ae^{-\alpha}, \tag{35}$$

from (20) and noting that (35) we get

$$f'=\frac{t_2 a}{t_2-t_1}-be^{\alpha}, \tag{36}$$

it is easy to see that $t_2=0$ by Lemma 1 and Lemma 4. From (35) and (36) we can obtain, respectively: $f'=-be^{\alpha}$, and $g=-ae^{-\alpha}$, i.e., $f'\cdot g'=ab$. This completes the proof of Theorem 2.

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分担两个值的亚纯函数的唯一性定理

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摘 要

本文主要讨论分担两个值的亚纯函数的唯一性问题，推广和改进了M. Ozawa、仪洪勋、C.C. Yang 等人的结果.

关键词: 亚纯函数，亏值，唯一性.