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L-fuzzy 拓扑空间中近良紧集的刻画

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摘　要

本文在L-fuzzy 拓扑空间中，针对L-fuzzy 子集定义了Mr-复盖及具有有限Mr-交性质的闭集族等概念，并以此刻画了近良紧集的一组特征，证明了近良紧性对正则闭集遗传以及是“L-good extension” 等结论.

关键词:　L-fuzzy 拓扑空间，近良紧集，fuzzy 格.

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Some Characterizations of Near N-compact Sets in L-fuzzy Topological Spaces *

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Abstract In this paper, the concepts of Mr-cover of L-fuzzy subsets in L-fuzzy topological spaces and the family of L-fuzzy subsets which has the finite Mr-intersection property are introduced and, some characterizations of near N-compact sets are given and, it is proved that the near N-compactness is hereditary for regular closed L-fuzzy subsets and that near N-compactness is an "L-good extension".

Keyworks L-fuzzy topological space, near N-compact L-fuzzy subset, fuzzy lattice.

The concepts and symbols used in this paper follows [1,3].

Definition 1 *Let (L^X,δ) be an L-fts, and $r \in p(L)$. $\Psi \subset \delta$ is called an Mr-cover of $A \in L^X$, if for each $x \in \tau_{r'}(A)$, there is $U \in \Psi$ such that $x \in \tau_r(U^{-0})$. Ψ is called an Mr^+-cover of A, if there is $t \in \alpha^*(r)$ such that Ψ is an Mt-cover of A.*

Definition 2 *Let (L^X,δ) be an $L-fts$, $A \in L^X$, $r \in p(L)$. $\Omega \subset L^X$ is called the family which has the finite r-(resp., r-) intersection property, or briefly, $f.Mr-i.p.$ (resp., $f.r-i.p.$) in A if for each $\Psi \in 2^{(\Omega)}$ there exists $x \in \tau_{r'}(A)$ such that $(\wedge\Psi^{0-})(x) \geq r'$ (resp., $(\wedge\Psi)(x) \geq r'$), and $r \in p(L)$.*

Theore 1 *Let (L^X,δ) be an $L-fts$, $A \in L^X$, then the following statements are equivalent:*

(1) A is near N-compact.

(2) For any $r \in p(L)$ and each Mr-cover Ω of A, there is $\Psi \in 2^{(\Omega)}$ such that Ψ is an Mr^+-cover of A.

(3) For any $r \in p(L)$ and each family $\Phi \subset \delta'$ which has $f.Mr-i.p.$ in A, there is $x \in \tau_{r'}(A)$ sucthat $(\wedge\Phi^{0-})(x) \geq r'$.

(4) For any $r \in (L)$ and each family Ω of regular closed L-fuzzy subsets which has $f.r-i.p.$ in A, there is $x \in \tau_{r'}(A)$ such that $(\wedge\Omega)(x) \geq r'$.

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Proof (1)$\Longrightarrow$(2). For any $r \in p(L)$, suppose Ω is any Mr-cover of A, then $\Phi = \Omega'$ is an $Mr' - RF$ of A. In fact, for each $x_{r'} \in A$, we see that $x \in \tau_{r'}(A)$, and then there is $U \in \Omega$ such that $x \in \tau_r(U^{-0})$. Put $P = U'$, then $P \in \Phi$ and $P^{0-} \in \eta(x_{r'})$. Hence Φ is indeed an $Mr' - RF$ of A. From Theorem 1.1 in [3], there is $\Psi \in 2^{(\Omega)}$ such that $\Theta = \Psi' \subset \Phi$ is an $M(r')^- - RF$ of A, i.e., there is $s \in \beta^*(r')$ such that Θ is an $Ms - RF$ of A. Put $t = s'$, then $t \in (\beta^*(r'))'$. For each $x \in \tau_{t'}(A) = \tau_s(A)$, x_s is a molecule in A, and then there is $P \in \Theta$ such that $P^{0-} \in \eta(x_s)$, and so $P'^{-0}(x) = P^{0-\prime}(x) \not\leq s' = t$. put $U = P'$, then $U \in \Psi$ and $x \in \tau_t(U^{-0})$. This shows that Ψ is an Mt-cover of A, and then Ψ is aMr^+-cover of A.

(2)$\Longrightarrow$(3). Suppose that (3) is untenable, then there exist $r \in p(L)$ and some $\Phi \subset \delta'$ which has $f.Mr - i.p.$ in A such that $(\wedge\Phi^{0-})(x) \not\geq r'$ holds for each $x \in \tau_{r'}(A)$, and thus $(\Phi^{0-\prime})(x) \not\leq r$. Hence there is $P \in \Phi$ such that $P'^{-0}(x) = P^{0-\prime}(x) \not\leq r$, i.e., $x \in \tau_r(P'^{-0})$, hence Φ' is an Mr-cover of A. From (2), there is $\Psi' = \{P_1', \cdots, P_n'\} \in 2^{(\Phi')}$ such that Ψ' is an Mr^+-cover of A, that is , there is $t \in \alpha^*(r)$ such that Ψ' is an Mt-cover of A, and thus for each $x \in \tau_{t'}(A)$ there is $P_i \in \Psi'$ such that $x \in \tau_t(P_i'^{-0})$, and so $(\vee\Psi'^{-0})(x) \not\leq t$, and hence $(\wedge\Psi^{0-})(x) = (\wedge\Psi'^{-0\prime})(x) \not\geq t'$. Since $t \in \alpha^*(r)$, $t' \leq r'$, and then $(\wedge\Psi^{0-})(x) \not\geq r'$, this contradicts that Φ has $f.Mr - i.p.$ in A.

(3)$\Longrightarrow$(4). It is easy.

(4)$\Longrightarrow$(1). Suppose that A is not near N-compact, then from Corollary of [3], there are $\alpha \in M(L)$ and some α-regular closed RF Φ of A, such that any $\Psi \in 2^{(\Phi)}$ is not α^--regular closed RF of A, i.e., for each $t \in \beta^*(\alpha)$ there exists molecule x_t in A such that $P \notin \eta(x_t)$ holds for each $P \in \Psi$, and so $t \leq (\wedge\Psi)(x)$. This shows that Φ has $f.t' - i.p.$ in A. From (4) there is $x \in \tau_t(A)$ such that $(\wedge\Phi)(x) \geq t$. From the arbitrariness of $t \in \beta^*(\alpha)$ and $\vee\beta^*(\alpha) = \alpha$ we have $(\wedge\Phi)(x) \geq \alpha$, this contradicts that Φ is an $\alpha - RF$ of A. Therefore A is near N-compact.

We have proved in [3] that near N-compactness is a "good extension", now further prove that it is an "L-good extension". For this we need the following two lemmas.

Lemma 1 *Let $(L^X, \omega_L(\tau))$ be the $L-fts$ induced by a crisp topological space (X,τ), $A \in L^X$. Then*

(1)[2] $\forall a \in L\backslash\{0\}$, $\tau_a(A^-) = \cap_{\alpha\in\beta(a)}(\tau_\alpha(A))^-$.

(2)[5] $A^0 = \vee_{r\in L} r\chi_{(\tau_r(A))^\circ}$.

Lemma 2 *Let $(L^X, \omega_L(\tau))$ be the $L-fts$ induced by a crisp topological space (X,τ), $a \in L\backslash\{0\}$, $A \in L^X$. Then*

(1) $\tau_a(A^-) = (\tau_a(A))^-$.

(2) $A^{-0} = \vee_{r\in L} r\chi_{(\tau_r(A))^{-\circ}}$.

(3) $\forall B \subset X$, $(\chi_B)^{-0} = \chi_{B^{-\circ}}$

Proof (1) From Lemma 1 (1) we have

$$\tau_a(A))^- \subset \tau_a(A^-))^- = \tau_a(A^-).$$

Conversely, if there is $x \in X$ such that $x \in \tau_a(A^-)$ but $x \notin (\tau_a(A))^-$, then there is a neighborhood W of x in (X,τ) such that $W \cap \tau_a(A)$ is empty, and hence $A(y) \not\geq a$ holds

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for each $y \in W$. It follows from $\vee\beta(a) = a$ that there is $\alpha \in \beta(a)$ such that $A(y) \not\geq \alpha$. Therefore $W \cap \tau_\alpha(A)$ also is empty, and so $x \notin (\tau_\alpha(A))^-$. On the other hand, by $x \in \tau_a(A^-)$ and Lemma 1 (1), $x \in (\tau_\alpha(A))^-$ holds for each $\alpha \in \beta(a)$. This is a contradiction. Hence $\tau_a(A^-) \subset (\tau_a(A))^-$.

(2) From Lemma 1 (2) and above (1) we get

$$A^{-0} = (A^-)^0 = \vee_{r\in L} r\chi_{(\tau_r(A^-))^0} = \vee_{r\in L} r\chi_{(\tau_r(A))^{-0}}.$$

(3) It is clear that $\tau_r(\chi_B) = B$ holds for each $r \in L\backslash\{0\}$. From above (2) we have

$$(\chi_B)^{-0} = \vee_{r\in L\backslash\{0\}} r\chi_{B^{-0}} = (\vee_{r\in L\backslash\{0\}} r)\chi_{B^{-0}} = \chi_{B^{-0}}.$$

Now we will prove that near N-compactness is an "L-good extension".

Theorem 2 *Let $(L^X, \omega_L(\tau))$ be the $L-$ fts induced by a crisp topological space (X, τ). Then $(L^X, \omega_L(\tau))$ is near N- compact iff (X, τ) is near compact.*

Proof Necessity: let Φ be any regular open cover of (X, τ), then $\Theta = \{\chi_A : A \in \Phi\} \subset \omega_L(\tau)$. Take arbitrarily $\alpha \in M(L)$, then $r = \alpha' \in p(L)$. It is clear that Θ is an Mr-cover of 1_x. From Theorem 1, there is $\Psi = \{\chi_{A_i} : i = 1, 2, \cdots, n\} \in 2^{(\Theta)}$ such that Ψ is an Mr^+-cover of 1_x, i.e., there is $t \in \alpha^*(r)$ such that Ψ is an Mt-cover of 1_x. From this we will deduce that $\Omega = \{A_i : i = 1, 2, \cdots, n\} \in 2^{(\Phi)}$ is the cover of (X, τ). In fact, for each $x \in X = \tau_{t'}(1_x)$, there is $A_i \in \Omega$ such that $x \in \tau_t((\chi_{A_i})^{-0})$, and so $(\chi_{A_i})^{-0}(x) = 1$. It follows from Lemma 2 (3) that $\chi_{A_i^{-0}}(x) = 1$, and so $x \in A_i^{-0} = A_i$. This shows that Ω is indeed a cover of (X, τ), and thus (X, τ) is near compact.

Sufficiency: let Φ be any Mr-cover of $(L^X, \omega_L(\tau))$, i.e., Φ be any Mr-cover of 1_x $(r \in p(L))$, then $\forall x \in X = \tau_{r'}(1_x)$, $\exists U_x \in \Phi$ such that $x \in \tau_r(U_X^{-0})$. From $\wedge\alpha^*(r) = r$ there is $t(x) \in \alpha^*(r)$ such that $x \in \tau_{t(x)}(U_X^{-0})$. By Lemma 2 (2) we have $(U_x^{-0})(x) = (\vee_{a\in L} a\chi_{(\tau_a(U_X))^{-0}})(x)$, and thus there is $a(x) \in L$ such that $x \in \tau_{t(x)}(a(x))\chi_{(\tau_{a(x)}(u_x))^{-0}}$ so $x \in (\tau_{a(x)}(U_x))^{-0}(x)$ and $a(x) \not\leq t(x)$. Hence $\Theta = \{(\tau_{a(x)}(U_x))^{-0} : x \in X\}$ is the regular open cover of (X, τ), and then there are $x_1, x_2, \cdots, x_n \in X$ such that $\Psi = \{(\tau_{a(x_i)}(U_{x_i}))^{-0} : i = 1, 2, \cdots, n\} \in 2^{(\Theta)}$ is the regular open cover of (X, τ). Notice that $a(x_i) \not\leq t(x_i)$, from $\alpha^*(r)$ is a lower directed set, there is $t \in \alpha^*(r)$ such that $t \leq t(x_i), i = 1, 2, \cdots, n$. Now $\forall x \in X = \tau_{t'}(1_x)$, $\exists i \leq n$ such that $x \in (\tau_{a(x_i)}(U_{x_i}))^{-0}$. But

$$((U_{x_i})^{-0})(x) = (\vee_{a\in L} a\chi_{(\tau_a(U_{x_i}))^{-0}})(x) \geq (a(x_i)\chi_{(\tau_{a(x_i)}(U_{x_i}))^{-0}}(x) = a(x_i) \not\leq t(x_i),$$

and thus $x \in \tau_{t(x_i)}((U_{x_i})^{-0})$, further $x \in \tau_t((U_{x_i})^{-0})$. This shows that $\Omega = \{U_{x_i} : i = 1, 2, \cdots, n\} \in 2^{(\Phi)}$ is an Mr^+-cover of 1_x hence $(L^X, \omega_L(\tau))$ is near N-compact.

The following theorem shows that near N-compactness is hereditary for the regular closed L-fuzzy sets, the proof is similar to Theorem 4.9 in [4].

Theorem 3 *Let A be near N-compact L-fuzzy set in $L-$ fts (L^X, δ), and $B \in L^X$ a regular closed L-fuzzy set. Then $A \wedge B$ is near N-compact.*

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