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双周期核奇异积分方程数值解法注记

李　星
(宁夏大学，银川 750021)

摘　要

本文提出了在现代工程，如岩石力学、混凝土力学及固体力学中需要解决但未解决的问题，即如下的双周期核及双准周期核奇异积分方程:

$$A_3(t_0)\omega(t_0)+\frac{B_3(t_0)}{2\pi i}\int_L\omega(t)[\varsigma(t-t_0)-\varsigma(t_0)]dt+\frac{C_3(t_0)}{2\pi i}\int_L\omega(t)K_3(t,t_0)dt=D_3(t_0)$$

和

$$A_4(t_0)\omega(t_0)+\frac{B_4(t_0)}{2\pi i}\int_L\omega(t)\varsigma(t-t_0)dt+\frac{C_4(t_0)}{2\pi i}\int_L\omega(t)K_4(t,t_0)dt=D_4(t_0).$$

的数值解法. 希望看到许多好的结果.

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Remark of Quadrature Methods for the Numerical Solutions of Singular Integral Equations with Doubly Periodic Kernal *

Li Xing
(Dept. of Math., Ningxia University, Yinchuan, P.R.C., China)

Abstract In this paper, we propose an unsetted problem. Hope to see some good results on the quadrature methods for the numerical solution of singular integral equations with doubly periodic kernal or quasi-periodic kernal.

Keywords quadrature method, singular integral equation, doubly periodic kernal, numerical solution

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The methods for the numerical solution of singular integral equations are more and more popular in engineering, especially in mechanics. There are many papers on it. F. Erdogen, G.D. Gupta, P.S. Theocaris, N.I. Ioakimidis, D.Elliott, Jianke Lu (Chienke Lu), Jinyuan Du and many other authors have given varied quadrature methods for the numerical solution of singular integral equations with Cauchy kernal $1/(t-t_0)$ (it is a non-periodic kernal) or Hilberte kernal $\mathrm{ctg}(t-t_0)$ (it is a periodic kernal) ([1]-[8]), for example, on the following forms of singular integral equations

$$A_1(t_0)\omega(t_0)+\frac{B_1(t_0)}{2\pi i}\int_L\frac{\omega(t)}{t-t_0}dt+\frac{C_1(t_0)}{2\pi i}\int_L\omega(t)K_1(t,t_0)dt=D_1(t_0)$$

or

$$A_2(t_0)\omega(t_0)+\frac{B_2(t_0)}{2\pi i}\int_L\omega(t)\mathrm{ctg}(t-t_0)dt+\frac{C_2(t_0)}{2\pi i}\int_L\omega(t)K_2(t,t_0)dt=D_2(t_0).$$

Here $\omega(t)$ is the unknown function, $A_j(t_0), B_j(t_0), C_j(t_0), D_j(t_0)$ $(j=1,2)$ are all known functions and $\in H$, $K_j(t,t_0)$ are both known kernals with weak singularity, in general case,

$$\begin{aligned}K_1(t,t_0) &= d[\frac{(t-t_0)}{t-t_0}]+K_1^0(t,t_0),\\ K_2(t,t_0) &= d\ln[\frac{\sin(t-t_0)}{\sin(t-t_0)}]+K_2^0(t,t_0).\end{aligned}$$

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L consists of simple smooth curves in the complex plane.

But, in fact, the doubly periodic problems are very common in rock mechanics, concrete mechanics, solid mechanics, especially in fracture mechanics ([9]- [10]). By complex variable method the prime problem can be changed into solving singular integral equation of the form ([11]-[13])

$$A_3(t_0)\omega(t_0)+\frac{B_3(t_0)}{2\pi i}\int_L \omega(t)[\varsigma(t-t_0)-\varsigma(t_0)]dt+\frac{C_3(t_0)}{2\pi i}\int_L \omega(t)K_3(t,t_0)dt=D_3(t_0)$$

or

$$A_4(t_0)\omega(t_0)+\frac{B_4(t_0)}{2\pi i}\int_L \omega(t)\varsigma(t-t_0)dt+\frac{C_4(t_0)}{2\pi i}\int_L \omega(t)K_4(t,t_0)dt=D_4(t_0).$$

Here $A_j(t_0), B_j(t_0), C_j(t_0), D_j(t_0)$ $(j=3,4)$ are all known functions, $\omega(t)$ is the unknown function, $\varsigma(t)$ is the Weierstrass ς function and

$$\varsigma(t+2\omega_k)=\varsigma(t)+2\eta_k, \quad \eta_k=\varsigma(\omega_k) \quad k=1,2.$$

So $\varsigma(t)$ is a quasi-periodic function, ω_k $(k=1,2)$ are the fundamental periods, kernal $[\varsigma(t-t_0)-\varsigma(t_0)]$ is a doubly periodic in t_0, kernal $\varsigma(t-t_0)$ is a quasi-periodic kernal, $K_j(t,t_0)$ are known kernals with weak singularity, in general case,

$$\begin{aligned} K_3(t,t_0) &= d\ln[\frac{\sigma(t-t_0)\sigma(t_0)}{\overline{\sigma(t-t_0)\sigma(t_0)}}]+K_3^0(t,t_0), \\ K_4(t,t_0) &= d\ln[\frac{\sigma(t-t_0)}{\overline{\sigma(t-t_0)}}]+K_4^0(t,t_0). \end{aligned}$$

Here $\sigma(t)$ is the Weierstrass σ function:

$$\varsigma(t)=\frac{\sigma'(t)}{\sigma(t)}.$$

However, there is no paper on the quadrature methods for the numerical solution of it. The author hopes to see some good results on it!

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李 星
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和

$$A_4(t_0)\omega(t_0)+\frac{B_4(t_0)}{2\pi i}\int_L \omega(t)\varsigma(t-t_0)dt+\frac{C_4(t_0)}{2\pi i}\int_L \omega(t)K_4(t,t_0)dt=D_4(t_0).$$

的数值解法. 希望看到许多好的结果.