Similar to the proof of Lemma 1, we can show that

$$\int_0^\infty \int_U |f_{t,j}(x)|dxt^{-1}dt \leq C2^{j(-s+n/2)}$$

and then obtain

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The estimates (30),(31),(33) and (34) give $\sigma \leq C$.

Similarly, we can prove $\sigma_3 \leq C$. These complete the proof for regular atoms. For any exceptional atom $a(x)$, let $a(x) = A(x) + c$, where $A(x) = a(x) - \int_G a(x)dx$ is a regular atom and $c = \int_G a(x)dx$. We easily check that $\|Tc\|_{B(G)} \leq C$. The estimate (15) is therefore proved.

A direct application of Theorem A is following theorem:

Theroem B *Suppose T is a multiplier operator associated to a multiplier $\{m(\lambda)\}_{\lambda\in\hat{G}}$. If $m(\lambda) = \|\lambda+\beta\|^{ia}$ for some $a \in \mathbb{R}$. Then T is bounded in Besov spaces.*

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Besov 空间上的 Hörmander 乘子定理

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摘　要

本文在紧 Lie 群上建立了一个 Besov 空间上的 Hormander 乘子定理.

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A Hörmander Multiplier Theorem on Besov Spaces *

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Abstract We set up a Hörmander multiplier theorem in the Besov spaces on a compact Lie group.

Keywords H^p spaces, multiplier, Besov spaces, compact Lie groups.

Classification AMS(1991) 43A15, 43A77/CCL O174.3

Notations

Let G be a connected, simply connected, compact semisimple Lie group of dimension n, and g be the Lie algebra of G. Then g can be identified with $T_e(G)$, the tangent space of G at the identity element e of G. Therefore, we can choose an orthonormal basis $X_1, \cdots, X_n$ of g. Let d be the bi-invariant metric on G, denote $d(x,e)$ by $|x|$ for $x \in G$.

An exceptional atom $a(x)$ is an L^∞ function satisfying:

$$\|a\|_\infty \le 1, \tag{1}$$

$$\|X_j a\|_\infty \le 1, \quad j = 1, 2, \cdots, n. \tag{2}$$

A regular $(1, \infty)$ atom is a function $a(x)$ supported in some ball $B(y, \rho)$ which satisfies:

$$\|a\|_\infty \le \rho^{-n}, \tag{3}$$

$$\|X_j a\|_\infty \le \rho^{-1-n}, \quad j = 1, 2, \cdots, n, \tag{4}$$

$$\int_G a(x) dx = 0. \tag{5}$$

The atomic Besov space $B_a(G)$, is the space of all $f \in L(G)$ having the form

$$f(x) = \sum c_k a_k(x) \tag{6}$$

with $\sum |c_k| < \infty$.

Where each $a(x)$ is either a regular atom or an exceptional atom. The "norm" $\|f\|_{B_a}$ is the infimum of all expressions $(\sum |c_k|)$ for which we have such a representation of $f(x)$.

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It is easy to see that the space B_a is a subspace of the Hardy space $H(G)$ which is defined in [4].

Let $\phi(\theta)$ be a radial function in $\Delta(I\!R^l)$ which satisfies:

$$\hat{\phi}(0) = 0, \tag{7}$$

$$\text{supp } \phi \subseteq \{\theta \in I\!R^l, |\theta| < 1\}, \tag{8}$$

$$\int_0^\infty (\hat{\phi}(s))^2 s^{-1} ds = c \neq 0. \tag{9}$$

Define a C^∞ central function on G by

$$\phi_t(x) = \sum_{\lambda\in\Lambda} \hat{\phi}(t\|\lambda+\beta\|) d_\lambda \chi_\lambda(x). \tag{10}$$

Then the S-functions of any function $f(x) \in L(G)$ is defined by

$$S_\Phi f(x) = \int_{\Gamma_x} |f * \phi_t(y)| t^{-n-1} dy dt, \tag{11}$$

where

$$\Gamma_x = \{(y,t), d(y,x) < t\}.$$

The Besov space $B(G) = \dot{B}_1^{0,1}(G)$ is the collection of all $f \in L(G)$ such that

$$\|f\|_{B(G)} = \|S_\Phi f\|_{L(G)} < \infty. \tag{12}$$

In our prior paper [5], it was proved that $\|f\|_{B_a} \simeq \|f\|_B$ hence $B(G) = B_a(G)$. Using this atomic characterization, we will prove here a Hörmander multiplier theorem on Besov space.

Throughout this paper, the letter C will denote (possibly different) constants that are independent of the essential variables in the argument, this independence will be clear from the context.

A Hõlmander Multiplic Theorem

Given a bounded multi-sequence $\{m(\lambda)\}_{\lambda\in\Lambda}, m(\lambda) \in C$, define the operator T on the space of finite linear combination of entry function on G by

$$(\widehat{Tf})(\lambda) = m(\lambda)\hat{f}(\lambda), \quad \lambda \in \Lambda.$$

Recall that difference operators δ^j are defined on sequences by

$$\delta^1(a_n) = a_{n+1} - a_n, \ \delta^{j+1}(a_n) = \delta^1(\delta^j(a_n)). \tag{13}$$

Given an l-tuple $J = (j_1, \cdots, j_l)$ of non-negative integers, the partial difference operator δ^J is defined analogously on multi-sequences $\{m(\lambda)\}_{\lambda\in\Lambda}$. The main result is the following Hörmander multiplier theroem:

Theorem A *Let s be the smallest even integer such that $s > n/2$. Suppose that $m \in L^\infty$ such that for l-tuple J with $j_1 + j_2 + \cdots + j_l = |J| \le s$ and all $R > 0$,*

$$\sum_{R \le |\lambda| \le 2R} |\delta^J m(\lambda)|^2 \le C R^{l-2|J|}. \tag{14}$$

Then T is a bounded operator on $B(G)$.

Proof By the atomic decomposition of Besov spaces, it is enough to prove that

$$\|Ta\|_{B(G)} \le C \tag{15}$$

uniformly for all exceptional and regular atoms $a(x)$.

For a regular $(1, \infty)$ atom $a(x)$, without loss of generality, we can assume that supp $a \subseteq B(e, \rho)$. By definition of Besov spaces,

$$\|Ta\|_{B(G)} \simeq \|S_\Phi(Ta)\|_{L(G)} \simeq \int_G \int_0^\infty |Ta * \Phi_t)(y)| dy t^{-1} dt,$$

where Φ is a function satisfying (7)–(9) and

$$\int_{R^l} \theta^I \Phi(\theta) d\theta = 0 \quad \text{for all multi-indices } I \text{ with } |I| \le n+1. \tag{16}$$

For this chosen function Φ, it is easy to see that for any $|I| \le n$,
(i) $\frac{\partial^I}{\partial \theta} \hat{\Phi}(\theta) = O(|\theta|)$ as $|\theta| \to 0$, and (ii) $\frac{\partial^I}{\partial \theta} \hat{\Phi}(\theta) = O(|\theta|^{-n})$ as $|\theta| \to \infty$.
Take radial functions $\eta, \psi \in C^\infty(I\!R^l)$ with

$$0 \le \eta \le 1, \eta(H) = 1 \text{ for } 1/2 \le |H| \le 2, \text{supp}\eta \subset \{\frac{1}{4} \le |H| \le 4\}; \tag{17}$$

$$0 \le \psi \le 1, \text{supp}\psi \subset \{\frac{1}{2} \le |H| \le 2\}, \sum_{j=-\infty}^{\infty} \psi(2^{-j}H) = 1. \tag{18}$$

Thus $\psi\eta = \psi$ and for any atom $a(x)$,

$$(Ta * \Phi_t)(x) = \sum_{j=-\infty}^{m} f_{t,j} * b_j(x) = \sum_{j=0}^{\infty} f_{t,j} * b_j(x), \tag{19}$$

where

$$f_{t,j}(x) = \sum_{\lambda \in \hat{G}} d_\lambda m(\lambda) \eta(\frac{|\lambda + \beta|}{2^j}) \hat{\Phi}(t|\lambda + \beta|) \chi_\lambda(x),$$

$$b_j(x) = \sum_{\lambda \in \hat{G}} d_\lambda \psi(\frac{|\lambda + \beta|}{2^j}) Tr(\hat{a}(\lambda) U_\lambda(x))$$

and thus $f_{t,j} = b_j = 0$ if $j < 0$. For these $f_{t,j}$'s and b_j's we have the following lemmas:

Lemma 1 *Let s be the smallest even integer such $s > n/2$. Then*

$$\int_0^\infty \|f_{t,j}\|_1 t^{-1}dt \le C, \tag{20}$$

$$\int_0^\infty \int_G |f_{t,j}(x)|^2[1+(2^j|x|)^2]^s dx t^{-1}dt \le C2^{jn}, \tag{21}$$

where $|x| = d(x,e)$ is the distance between x and e.

Proof First we introduce a function $r(x)$ which was defined in [7]. Let [w] be the order of the Weyl group. For $x \in G, x$ conjugate to a point $\exp(\xi^{-1}\tau), \tau \in t^*$, we set $r(x) = (\sum_{w\in W} e^{i(w(\beta),\tau)}) - |W|$. Then $r(x) \sim |x|$ (see Lemma 9 of [7]). Now by Hölder's inequality

$$\|f_{t,j}\|_1 \le \{\int_G |f_{t,j}(x)|^2[1+(2^j|x|^2]^s dx\}^{1/2}\{\int_G [1+(2^j|x|)^2]^{-s}dx\}^{1/2} = I_{t,1} \times I_2.$$

By Lemma 9 of [7],

$$\begin{aligned}
(I_{t,1})^2 &= \sum_{k=0}^{s} C_k \int_G (2^j|x|)^{2k}|f_{t,j}(x)|^2 dx \le \sum_{k=0}^{s} C_k(2^j)^{2k}\int_G r(x)^k|f_{t,j}(x)|^2 dx \\
&= \sum_{k=0}^{s} C_k(2^j)^{2k}\|r^{k/2}f_{t,j}\|_2^2 \\
&\le C\sum_{k=0}^{s} C_k 2^{2kj} \sum_{|N|+|I|+|M|+|L|=k}\ \sum_{2^{j-1}<|\lambda+\beta|<2^{j+1}} [\delta^I(m(\lambda)) \\
&\quad \cdot\delta^L(|\lambda|^{(n-l)/2})\delta^M(\eta(\frac{|\lambda+\beta|}{2^j}))\delta^N(\hat{\Phi}(t|\lambda+\beta|))]^2.
\end{aligned}$$

Now $\int_0^\infty (I_{t,1}^2)t^{-1}dt = \int_0^{2^{-j}}(I_{t,1})^2 t^{-1}dt + \int_{2^{-j}}^\infty (I_{t,1})^2 t^{-1}dt = A + B$.

Using (i) to estimate the term A, we have

$$A \le C\sum_{k=0}^{s} C_k(2^j)^{2k+n-l-2(|N|+|L|+|M|-l+|I|)} \le C2^{jn}.$$

Applying (ii) to estimate the term B, we have

$$\begin{aligned}
B &\le C\sum_{k=0}^{s} C_k 2^{2jk} \sum_{|I|+|M|+|L|+|N|=k} ((2^j)^{(n-l)/2-|L|-|M|-|N|-1})^2 \\
&\quad \times \sum_{2^{j-1}\le|\lambda+\beta|\le 2^{j+1}} (\delta^I(m(\lambda))^2 \int_{2^{-j}}^\infty t^{-3}dt \le C2^{jn}.
\end{aligned}$$

These prove (21). Similarly, $\int_0^\infty I_{t,1}t^{-1}dt \le C2^{jn/2}$. But it is easy to check that $(I_2)^2 \le C2^{-jn}$. So Lemma 1 is proved.

Lemma 2 *Let $a(x)$ be a regular $(1,\infty)$ atom with supp $a \subset B(e,\rho)$ and $r \in N$ be any fixed integer. Suppose that $i_0 \in N$ such that 2^{i_0} is sufficiently large. Then for $j \geq i_0$,*

$$|b_j(x)| \leq \begin{cases} C\rho^n 2^{j(n+1-r)}(d(x,e)^{-r} + \Delta^{(j)}(x)^{-1}), & \text{if } d(x,e) > 2^{-j}+\rho, \\ C\rho^{n+1}2^{j(n+1)}, & \text{if } d(x,e) < 2^{-j}-\rho, \\ C\rho^{j(n-r)}[d(x,e)^{-r} + M(\Delta^{(j)-1})(x)], & \text{whenever } 2^j\rho > 1 \text{ and } d(x,e) > 2\rho, \end{cases}$$

where,

$$\Delta^{(j)}(x) = \Delta^{(j)}(\exp H) = \prod_{\alpha\in A} \sup(2^{-j}, \sin\alpha(H)/2) \tag{22}$$

and $M(\Delta^{(j)-1})$ is the Hardy-Littlewood maximal function of $(\Delta^{(j)})^{-1}$.

Proof Observe that $b_j = a * \Psi_j$ where

$$\Psi_j(x) = \sum_{\lambda\in\Lambda} d_\lambda \psi(\frac{|\lambda+\beta|}{2^j})\chi_\lambda(x). \tag{23}$$

A similar argument to the proof of (6.2) in [2] proves this Lemma.

Now we are ready to prove the inequality (15) for any $(1,\infty)$ atom. Take a positive integer $j_0 = j_0(\rho)$ such that $2^{j_0}\rho \leq \varepsilon_0/4 < 2^{j_0+1}\rho$. Here ε_0 is a positive number lying in the interval $(0,1)$ such that $\exp^{-1} L_{x^{-1}}$ is an analytic diffeomorphism of $B(x,\varepsilon_0)$ onto $B(0,\varepsilon_0)$, a ball centered at the origin of g(L_x is the left translation by x). Put

$$U_0 = \{x \in G : d(x,e) \leq 2\rho\},$$

$$U_k = \{x \in G : 2^k\rho \leq d(x,e) < 2^{k+1}\rho\},$$

for $k = 1,2,\cdots,j_0$ and $U = \{x \in G : d(x,e) > 2^{j_0-2}\rho\}$. By definition,

$$\|Ta\|_B \leq C\sum_{k=0}^{j_0(\rho)}(\int_0^\infty(\int_{U_k}|Ta*\Phi_t(x)|dx + \int_U |Ta*\Phi_t(x)|dx)t^{-1}dt).$$

Fix an $i_0 \in N$ so that 2^{i_0} is sufficiently large. Then we need only show:

$$\sigma = \sum_{k=i_0}^{j_0(\rho)}\int_0^\infty\int_{U_k}|Ta*\Phi_t(x)|dxt^{-1}dt \leq C, \tag{24}$$

and

$$\int_0^\infty\int_U |Ta*\Phi_t(x)|dxt^{-1}dt \leq C \tag{25}$$

with C independent of a. We will only prove (22), the proof of (23) can be completed in a similar way. By (19) we have

$$\sigma \leq \sum_{i=1}^{3}\sigma_i,$$

where

$$\sigma_1 = \sum_{k=i_0}^{j_0(\rho)} \sum_{j=0}^{i_0-1} \int_0^\infty \int_{U_k} |f_{t,j} * b_j(x)| dx t^{-1} dt, \tag{26}$$

$$\sigma_2 = \sum_{j=i_0}^{j_0(\rho)} \sum_{k=i_0}^{j_0(\rho)} \int_0^\infty \int_{U_k} |f_{t,j} * b_j(x)| dx t^{-1} dt, \tag{27}$$

$$\sigma_3 = \sum_{j=j_0+1}^{\infty} \sum_{k=i_0}^{j_0(\rho)} \int_0^\infty \int_{U_k} |f_j * b_j(x)| dx t^{-1} dt. \tag{28}$$

We may assume that $\rho \leq 1$. Observe that $\|X^J \Psi_j\|_\infty \leq C$ for $j \leq i_0$. As a result we have $\|b_j\|_\infty \leq C$ and then, by Lemma 1, $\sigma_1 \leq C$. For σ_2 we observe that $i_0 \leq j \leq j_0(\rho)$. Hence $2^j \rho < 1$. Let $\tilde{U}_k$ be union of U_{k-1}, U_k and U_{k+1}. Then for $x \in U_k$

$$\begin{aligned} |f_{t,j} * b_j(x)| &\leq \sum_{l=0}^{k-2} \int_{U_l} |f_{t,j}(y) b_j(y^{-1}x)| dy + \int_{\tilde{U}_k} |f_{t,j}(y) b_j(y^{-1}x)| dy \\ &\quad + \sum_{l=k+2}^{j_0(\rho)} \int_{U_l} |f_{t,j}(y) b_j(y^{-1}x)| dy + \int_U |f_{t,j}(y) b_j(y^{-1}x)| dy \\ &= \sum_{i=1}^{4} I_i(x,t). \end{aligned}$$

$$\int_{U_k} I_1(x,t) dx \leq \sum_{l=0}^{k-2} \int_{U_l} |f_{t,j}(y)| \left(\int_{U_k} |b_j(y^{-1}x)| dx \right) dy. \tag{29}$$

Notice that $y \in U_l (l \leq k-2)$ and $x \in U_k$ imply that $d(y^{-1}x, e) > \rho 2^k$. We break the estimate of (29) into the following two cases:

Case 1 $2^k \rho > 2^{-j}$. In this case by Lemma 2 we have

$$\int_{U_k} |b_j(y^{-1}x)| dx \leq C(2^j \rho) 2^{j(n-r)} (2^k \rho)^{-r+n}$$

Put $k_0 = |\ln(2^j \rho)| / \ln 2$. Then $2^{k_0} \sim (2^j \rho)^{-1}$ and

$$\sum_{\substack{k=i_0 \\ 2^k \rho > 2^{-j}}}^{j_0} (2^k \rho)^{n-r} \leq C \sum_{k=k_0}^{j_0(\rho)} (2^k \rho)^{n-r} \leq C 2^{j(r-n)}$$

Thus by Lemma 1,

$$\sum_{j=i_0}^{j_0(\rho)} \sum_{\substack{k=i \\ 2^k \rho > 2^{-j}}}^{j_0(\rho)} \int_0^\infty \int_{U_k} I_1(x,t) dx t^{-1} dt \leq C \sum_{j=i_0}^{j_0(\rho)} (2^j \rho) \leq C.$$

Case 2 $2^k\rho \le 2^{-j}$. In this case, by Lemma 2 we have

$$\int_{U_k} |b_j(y^{-1}x)|dx = \int_{U_k\cap\{d(y^{-1}x,c)\le 2^{-j+1}\}} + \int_{U_k\cap\{d(y^{-1}x,c)>2^{-j+1}\}}$$
$$\le C\rho 2^{j(n+1)}(2^{-j})^n + C\rho(2^j)^{n+1-r}(2^{-j})^{-r+n} \le C(2^j\rho).$$

Thus,

$$\sum_{\substack{k=i_0\\ 2^k\rho\le 2^{-j}}}^{j_0(\rho)} 1 \le \sum_{k=i_0}^{k_0}(2^k\rho)^{-1/2}\sqrt{2}^{-j} \le C\rho^{-1/2}\sqrt{2}^{-j}.$$

Therefore, we have

$$\sum_{j=i_0}^{j_0(\rho)}\sum_{\substack{k=i_0\\ 2^k\rho\le 2^{-j}}}^{j_0(\rho)}\int_0^\infty\int_{U_k} I_1(x,t)dxt^{-1}dt \le C$$

and

$$\sum_{j=i_0}^{j_0(\rho)}\sum_{k=i_0}^{j_0(\rho)}\int_0^\infty\int_{U_k} I_1(x,t)dxt^{-1}dt \le C. \tag{30}$$

Similarly,

$$\sum_{j=i_0}^{j_0(\rho)}\sum_{k=i_0}^{j_0(\rho)}\int_0^\infty\int_{U_k} I_3(x,t)dxt^{-1}dt \le C, \tag{31}$$

$$\int_{U_k} I_2(x,t)dt \le \int_{\tilde{U}_k}|f_{t,j}(y)|(\int_{U_k}|b_j(y^{-1}x)|dx)dy.$$

Also by Lemma 2 we have $\int_{U_k}|b_j(y^{-1}x)|dx \le C(2^j\rho)$ and by Lemma 1, we have

$$\int_0^\infty\int_{\tilde{U}_k}|f_{t,j}(y)|dyt^{-1}dt \le C2^{j(-s+n/2)}(2^k\rho)^{-s+n/2} \tag{32}$$

Thus for $2^k\rho > 2^{-j}$

$$\sum_{j=i_0}^{j_0(\rho)}\sum_{\substack{k=i_0\\ 2^k\rho>2^{-j}}}^{j_0(\rho)}\int_0^\infty\int_{U_k} I_2(x,t)dxt^{-1}dt$$
$$\le C\sum_{j=i_0}^{j_0(\rho)}(2^j\rho)2^{j(n-s-n/2)}\sum_{k=k_0}^{j_0(\rho)}(2^k\rho)^{(s-n-n/2)} \le C.$$

For $2^k\rho \le 2^{-j}$ the same estimate can be obtained by using the fact $\int_0^\infty \|f_{t,j}\|_1 t^{-1}dt \le C$ instead of (32). Therefore,

$$\sum_{j=i_0}^{J_0(\rho)}\sum_{k=i_0}^{j_0(\rho)}\int_0^\infty\int_{U_k} I_2(x,t)dxt^{-1}dt \le C. \tag{33}$$

Similar to the proof of Lemma 1, we can show that

$$\int_0^\infty \int_U |f_{t,j}(x)| dx t^{-1} dt \le C 2^{j(-s+n/2)}$$

and then obtain

$$\sum_{j=i_0}^{j_0(\rho)} \sum_{k=i_0}^{j_0(\rho)} \int_0^\infty \int_{U_k} I_4(x,t) dx t^{-1} dt \le C. \tag{34}$$

The estimates (30),(31),(33) and (34) give $\sigma \le C$.

Similarly, we can prove $\sigma_3 \le C$. These complete the proof for regular atoms. For any exceptional atom $a(x)$, let $a(x) = A(x) + c$, where $A(x) = a(x) - \int_G a(x)dx$ is a regular atom and $c = \int_G a(x)dx$. We easily check that $\|Tc\|_{B(G)} \le C$. The estimate (15) is therefore proved.

A direct application of Theorem A is following theorem:

Theroem B *Suppose T is a multiplier operator associated to a multiplier $\{m(\lambda)\}_{\lambda\in\hat{G}}$. If $m(\lambda) = \|\lambda+\beta\|^{ia}$ for some $a \in \mathbb{R}$. Then T is bounded in Besov spaces.*

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Besov 空间上的 Hörmander 乘子定理

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摘　要

本文在紧 Lie 群上建立了一个 Besov 空间上的 Hormander 乘子定理.