Journal of Mathematical Research & Exposition
Vol. 15, Supp, Nov. 1995

Wighted Approximation by Baskakov-Durrmeyer Operators*

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Keywords Baskakov-Durrmeyer operators, weighted approximation, characterization.

Classification AMS(1991) 41A35/CCL O174.41

Let $D_n(f;x)=\sum_{k=0}^{\infty}v_{nk}(x)(n-1)\int_0^{\infty}v_{nk}(t)f(t)dt$ are the Baskakov-Durrmeyer operators, in [1], M. Heilmann investgated the L_p-approximation by a class of the linear combination of $D_n(f;x)$ and obtained the direct and inverse results. In this paper we will extend these results to weighted approximation and obtained following characterization theorems using the equivalence between K-functional and Ditian—Totik moduli of smoothness.

Theorem 1 *If* $\omega f\in L_p[0,\infty,1\leqslant p\leqslant\infty,-\frac{1}{p}<a<1-\frac{1}{p}$, β *is arbitary*, $\omega(x)=x^a(1+x)^\beta, x\in[0,\infty)$, *and* $0<r<1$, *then the following statements are equivalence:*

(A) $\|\omega(D_n(f)-f)\|_p=O(n^{-r})$;

(B) $K(f;t^2)=O(t^{2r})$

(C) $\|\omega\Delta_{t\varphi}^2(f)\|_{L_p[2t^2,\infty)}=O(t^{2r})$,

where K-functional

$$K(f;t^2):=\inf_{g\in D}\{\|\omega(f-g)\|_p+t^2\|\omega\varphi^2g''\|_p\},$$

$$D=\{f\,|\,\omega f\in L_p[0,\infty),f\in \mathrm{A.C.loc},f,f^1,\omega\varphi^2f''\in L_p[0,\infty)\},$$

$$\varphi^2(x)=x(1+x),x\in[0,\infty),\Delta_{t\varphi}^2(f)=f(x-t\varphi(x))-2f(x)+f(x+t\varphi(x)).$$

Proof From the following lemmas and a result of A. Grundmann[2], we have (A)$\Leftrightarrow$(B), and from [3] we have (B)$\Leftrightarrow$(C), the proof is completed.

Followed the method of [4], we have following lemmas:

Lemma 1 *For* $\omega f\in L_p[0,\infty),1\leqslant p\leqslant\infty$, *then we have* $\|\omega(D_n(f))\|_p\leqslant c\|\omega f\|_p$, *where* c *denote a constant (in what follows, c always denote different constant)*

Lemma 2 *For* $f\in D,1\leqslant p\leqslant\infty$, *then we have*

$$\|\omega\varphi^2D_n''(f)\|_p\leqslant c\|\omega\varphi^2f''\|_p.$$

Lemma 3 *For* $\omega f\in L_p[0,\infty),1\leqslant p\leqslant\infty$, *then we have*

* Received May. 5, 1992. Supported by the Natural Science Foundation of Zhejiang Province.

$$\| \omega\varphi^2 (D_n(f))'' \|_p \leqslant cn \| \omega f \|_p.$$

Lemma 4 *For $f \in D, 1 \leqslant p \leqslant \infty$, then we have*

$$\| \omega(D_n(f))' \|_p \leqslant c(\| \omega\varphi \|_p + \| \omega\varphi^2 f'' \|_p).$$

Lemma 5 *For $f \in D, 1 \leqslant p \leqslant \infty$, and $E_n = [\frac{1}{n}, \infty)$, then we have*

$$\| \omega(D_n(f) - f \|_{L_p(E_n)} \leqslant \frac{c}{n}(\| \omega\varphi \|_p + \| \omega\varphi^2 f'' \|_p).$$

Lemma 6 *For $f \in D, 1 \leqslant p \leqslant \infty$, then we have*

$$\| \omega f D_n(t-x)x \|_p \leqslant cn^{-1}(\| \omega f \|_p + \| \omega\varphi^2 f'' \|_p).$$

Lemma 7 *For $f \in D, 1 \leqslant p \leqslant \infty$, then we have*

$$\| \omega D_n(R_2(f,t,x);x) \|_p \leqslant cn^{-1} \| \omega\varphi^2 f'' \|_p.$$

where $R_2(f,t;x) = \int_x^t (t-v)f''(v)dv$.

References

[1] M. Heilmann, *Direct and converse results for operators of Baskakov-Durrmeyer type*, A.T.A., **5**:1(1989), 105—127.

[2] A. Grundmann, *Inverse Theorems for Kantorovich Polynomials, in "Fourier analysis and application theory"*, North Holland, Amsterdam, 1979, 395—401.

[3] Z. Ditian and V. Totik, *Moduli of smoothness*, New York, 1987.

[4] Xuan Peicai, *Weithted approximation by the linear combination of Baskakov Durrmeyer operators in L_p*, J. Engin. Math., **11**:2(1994), 53—68.